

ATRANS

**Analytical Solutions for
Three-Dimensional Solute Transport
from a Patch Source**

Appendix B: Test Problems

Overview

The ATRANS solutions have been checked extensively. For the purposes of demonstration, we describe the results from three sets of tests.

Test	Inflow Concentration History	Solutions
1	Constant	ATRANS1 ATRANS2 ATRANS3 ATRANS4
2	Finite duration step	ATRANS3 ATRANS4
3	Exponential-decay	ATRANS2 ATRANS3 ATRANS4

1. Test 1: Continuous Source

- Partially penetrating source
 - Constant concentration of patch along influent boundary
- Comparison with analytical solution of E.A. Sudicky
- Comparison with numerical solution (MT3D)

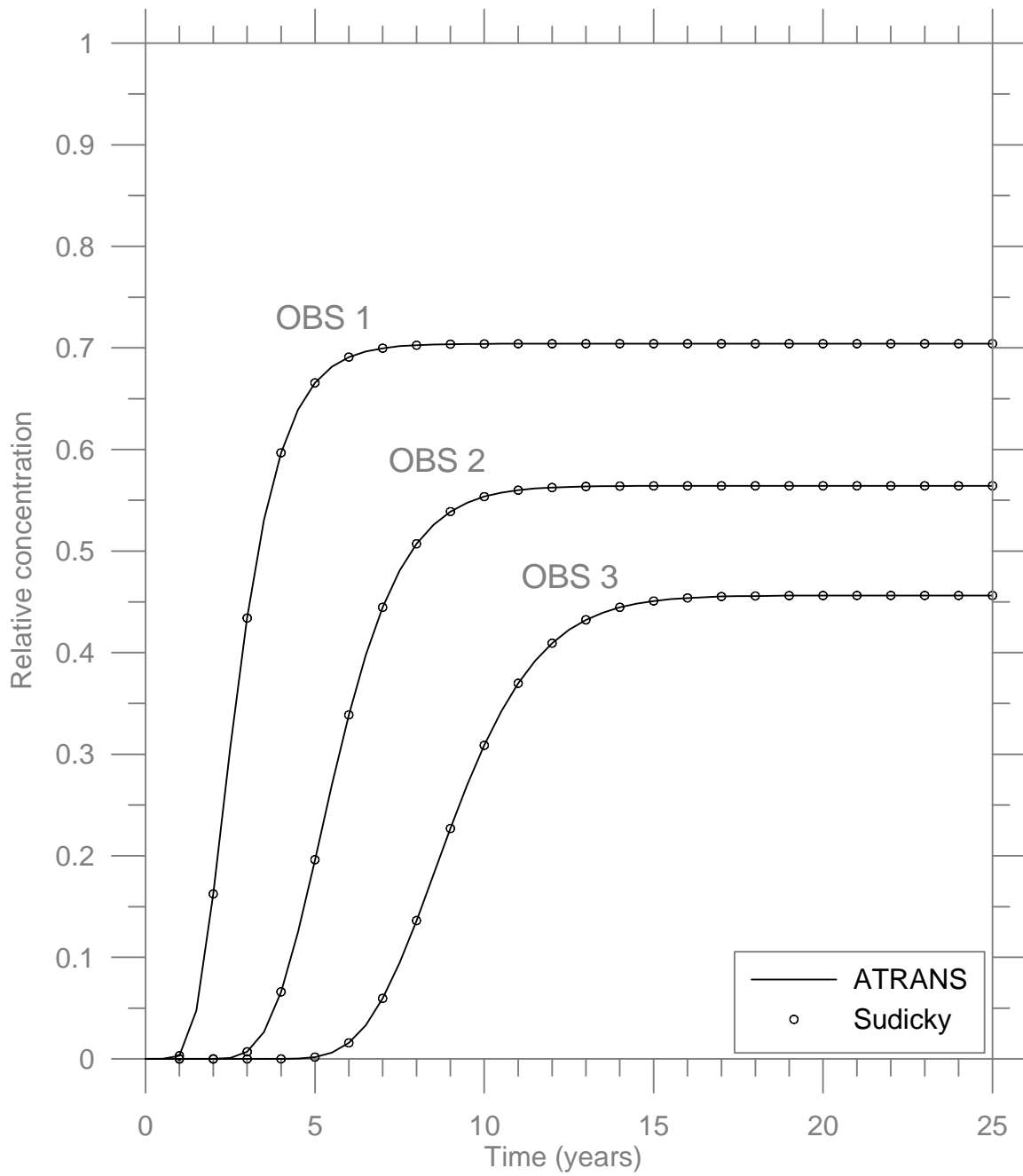
Input parameters:

Velocity, v	10.0 m/yr
Longitudinal dispersivity, α_L	2.5 m
Horizontal transverse dispersivity, α_{TH}	0.25 m
Vertical transverse dispersivity, α_{TV}	0.025 m
Effective diffusion coefficient, D^*	0.0 m ² /yr
Aquifer thickness, B	5.0 m
Decay coefficient, λ	0.0 yr ⁻¹
Retardation factor, R	1.0
Source width, $2 y_0$	25.0 m
Bottom of source, z_1	3.5 m
Top of source, z_2	5.0 m
Patch concentration, c_0	1.0

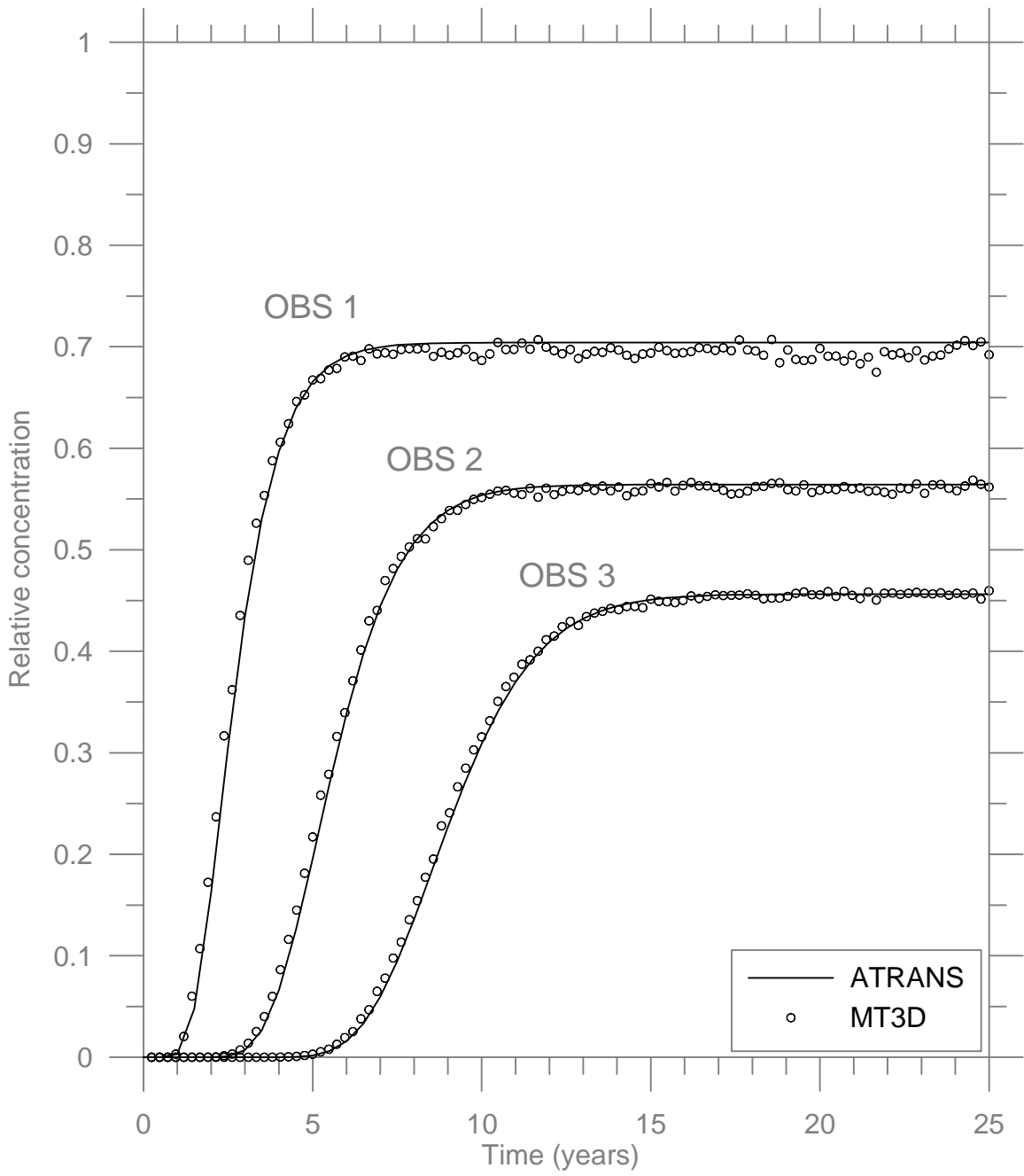
Results:

1. Breakthrough curves along plume centreline:
at $(x=30, y=0, z=4.25)$; $(x=60, y=0, z=4.25)$; $(x=95, y=0, z=4.25)$

Comparison with Sudicky analytical solution

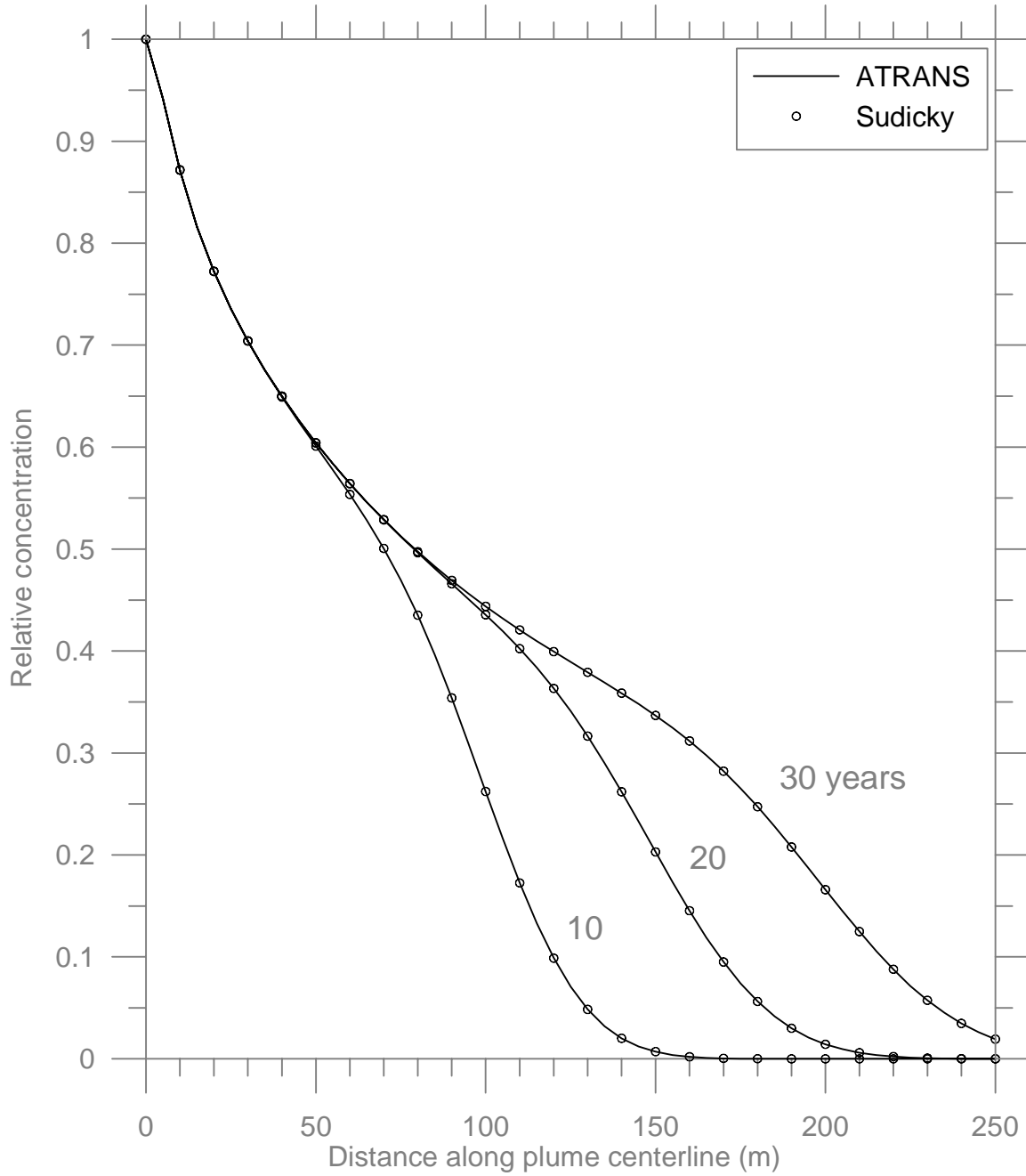


Comparison with MT3D numerical solution

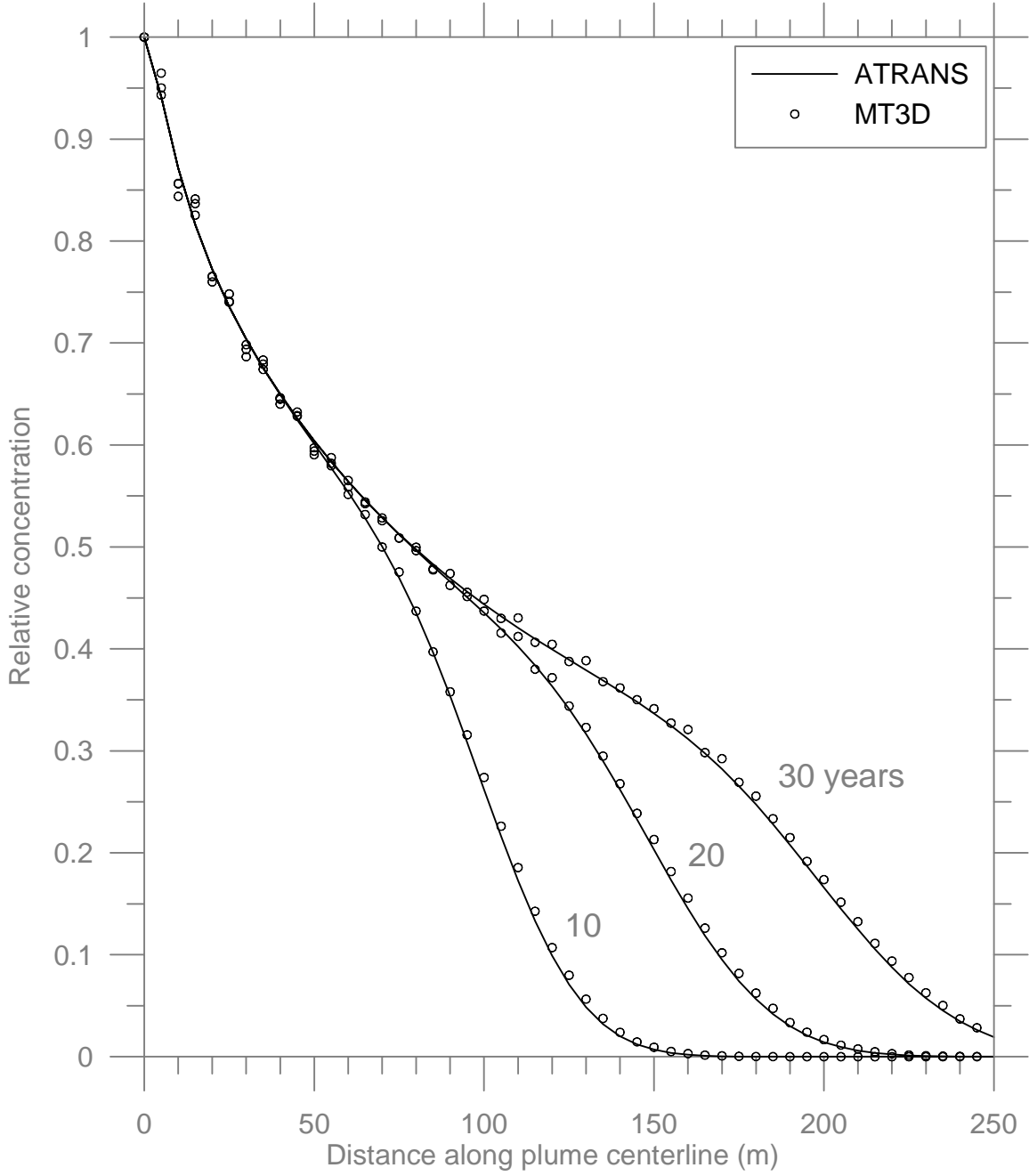


2. Longitudinal profiles along plume centreline:
 $y = 0, z = 4.25$ at times $t = 10, 15, 20$ years

Comparison with Sudicky analytical solution



Comparison with MT3D numerical solution



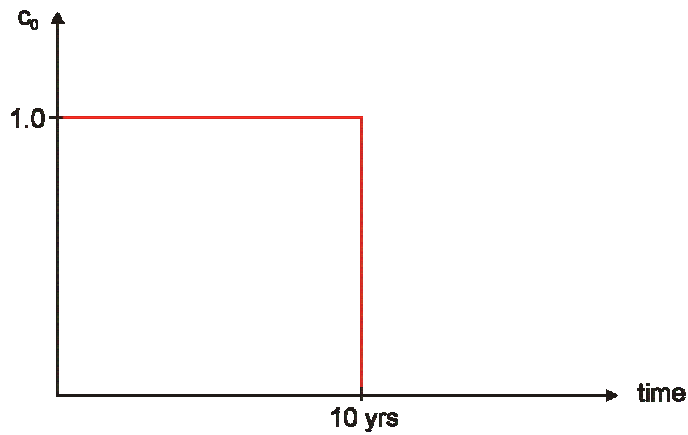
2. Test 2: Finite Duration Source

- Partially penetrating source
 - Transient concentration of patch along influent boundary
- Comparison with numerical solution (MT3D)

Input parameters:

Velocity, v	10.0 m/yr
Longitudinal dispersivity, α_L	2.5 m
Horizontal transverse dispersivity, α_{TH}	0.25 m
Vertical transverse dispersivity, α_{TV}	0.025 m
Effective diffusion coefficient, D^*	0.0 m ² /yr
Aquifer thickness, B	5.0 m
Decay coefficient, λ	0.0 yr ⁻¹
Retardation factor, R	1.0
Source width, $2 y_0$	25.0 m
Bottom of source, z_1	3.5 m
Top of source, z_2	5.0 m

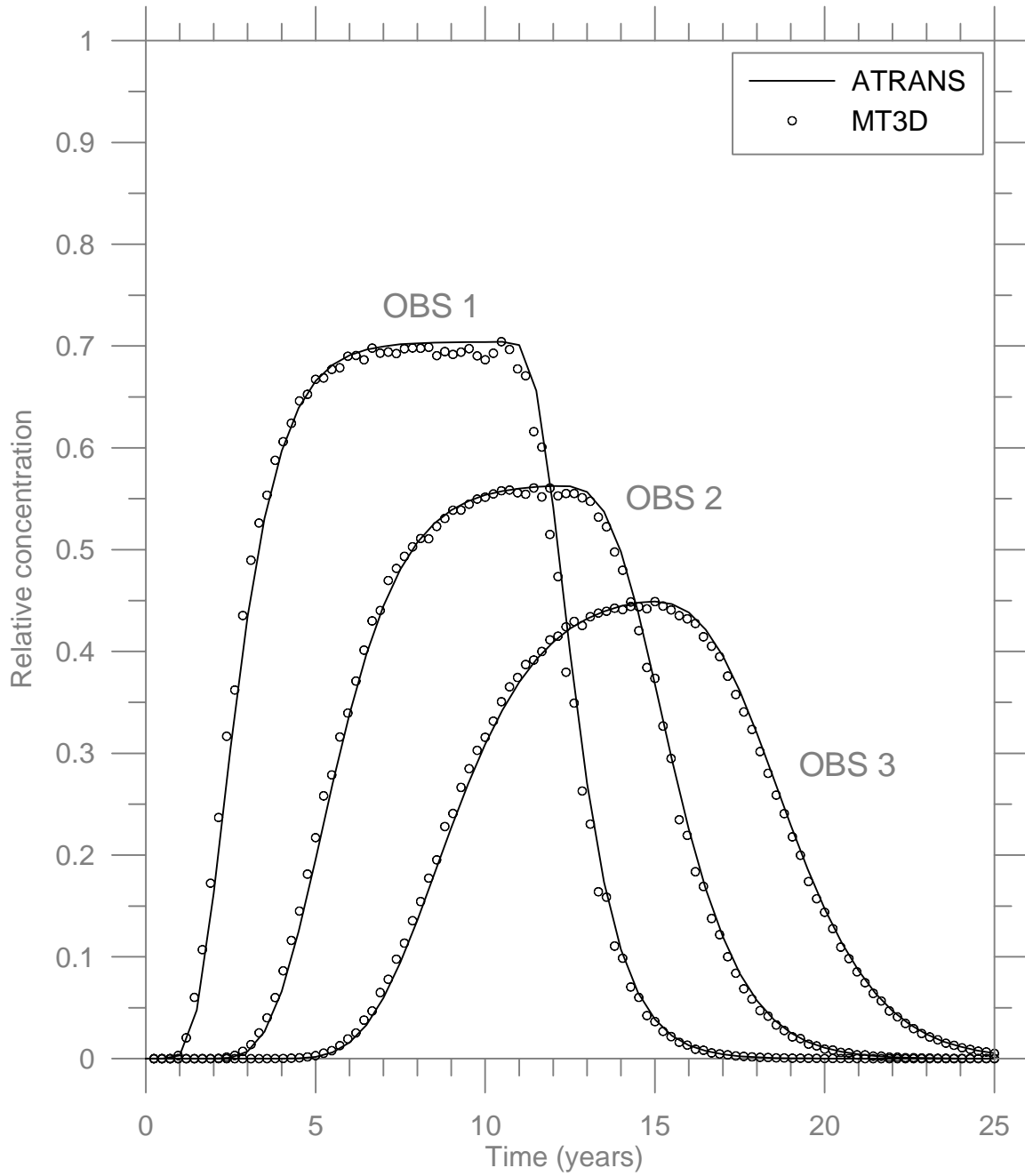
Patch concentration history:



Results:

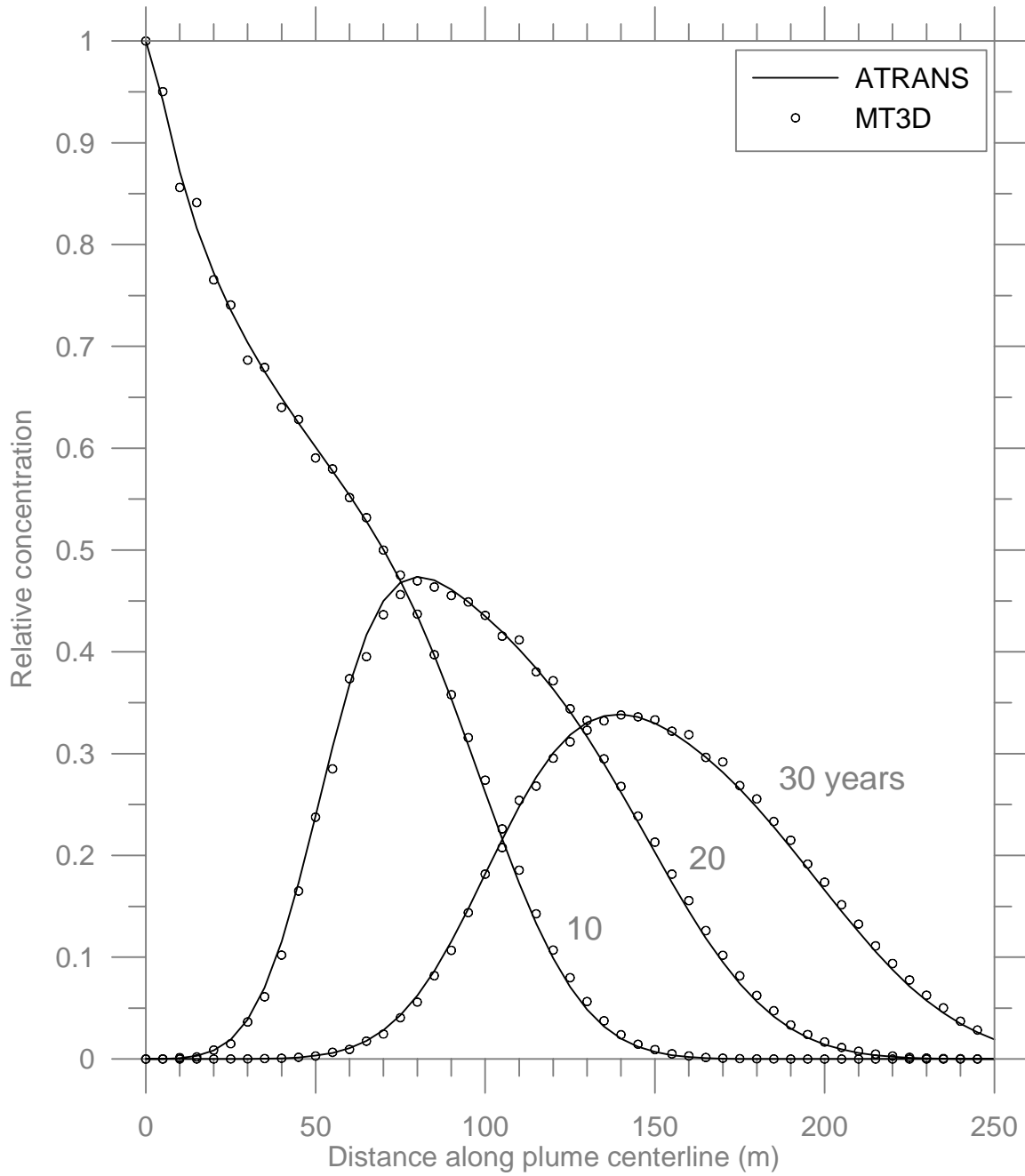
1. Breakthrough curves along plume centreline:
at $(x=30, y=0, z=4.25)$; $(x=60, y=0, z=4.25)$; $(x=95, y=0, z=4.25)$

Comparison with MT3D numerical solution



2. Longitudinal profiles along plume centreline:
 $y = 0, z = 4.25$ at times $t = 10, 15, 20$ years

Comparison with MT3D numerical solution



3. Test 3: Decaying Source

- Partially penetrating source
 - Decaying concentration of patch along influent boundary
- Comparison with analytical solution of E.A. Sudicky

Input parameters:

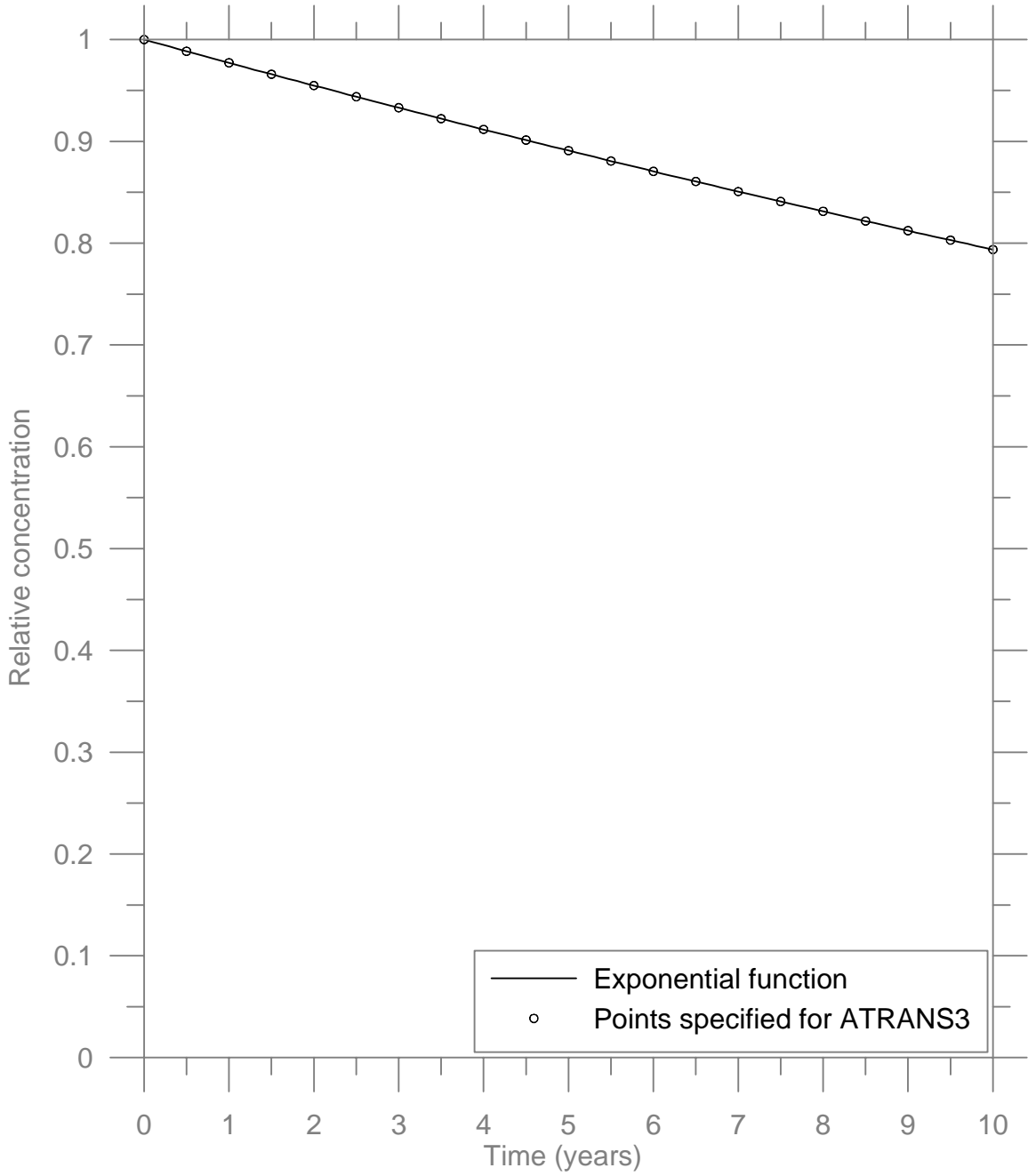
Velocity, v	10.0 m/yr	
Longitudinal dispersivity, α_L	1.0 m	
Horizontal transverse dispersivity, α_{TH}	0.01 m	
Vertical transverse dispersivity, α_{TV}	0.005 m	
Effective diffusion coefficient, D^*	0.0 m ² /yr	
Aquifer thickness, B	7.2 m	
Decay coefficient, λ	0.1386 yr ⁻¹	$\left[t_{\frac{1}{2}} = 5.0 \text{ yr} \right]$
Retardation factor, R	1.0	
Source width, $2 y_0$	10.0 m	
Bottom of source, z_1	5.8 m	
Top of source, z_2	6.6 m	

Concentration history of patch:

$$c_0(t) = c_0 \exp\{-\gamma t\}$$

$$\text{with } c_0 = 1.0; \gamma = 0.231 \text{ yr}^{-1} \left[t_{1/2} = 30 \text{ yrs} \right]$$

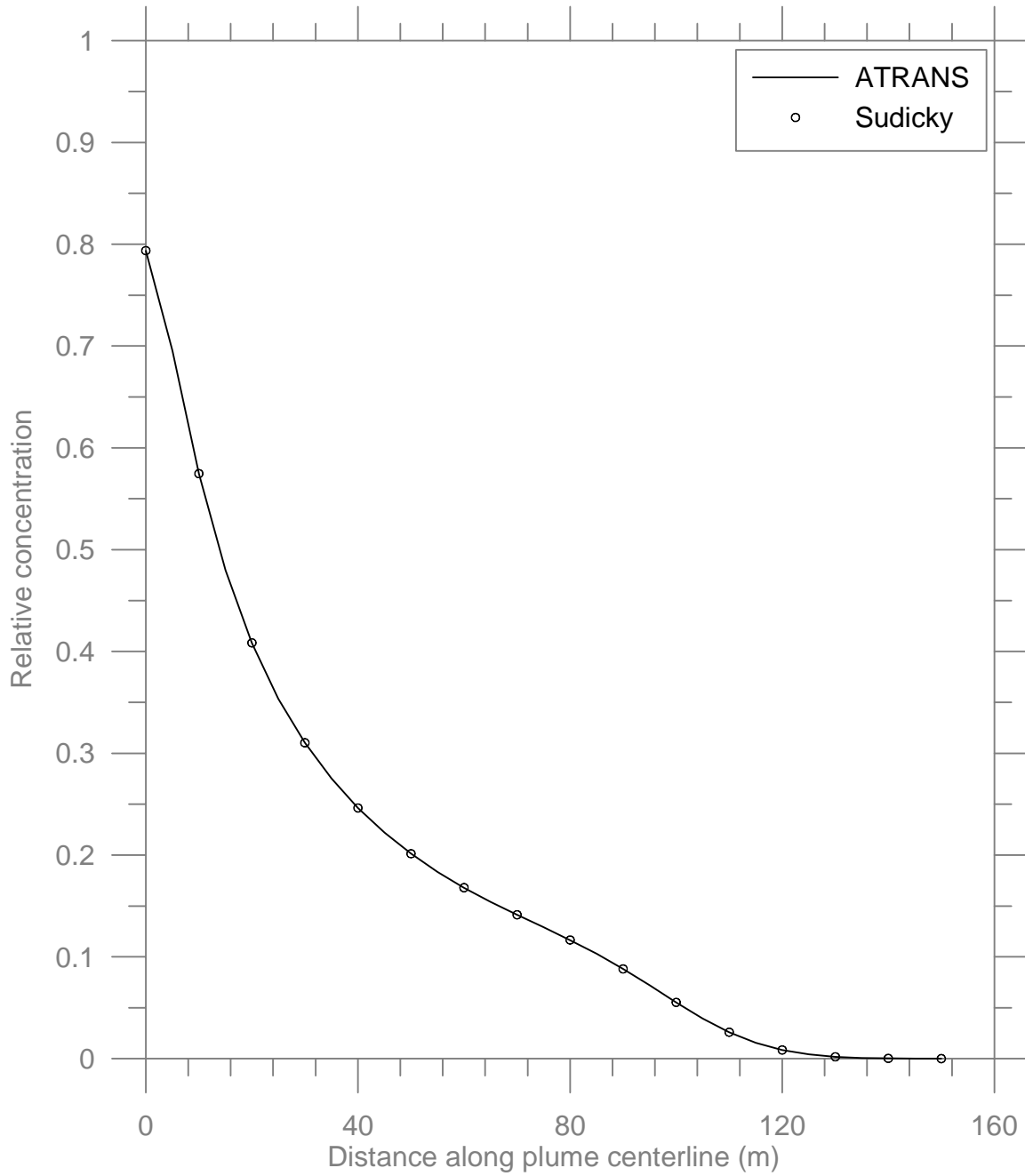
Specified inflow concentration history



Results:

Longitudinal profiles along plume centreline:
 $y = 0, z = 4.25$ at times $t = 10$ years

Comparison with Sudicky analytical solution



Longitudinal profiles along plume centreline:
 $y = 0, z = 4.25$ at times $t = 10$ years
Re-plotted with \log_{10} concentration axis

