WASHINGTON STATE
DEPARTMENT OF
ECOLOGY

Washington State Department of Ecology
Toxics Cleanup Program

Statistical Guidance for Ecology Site Managers

August, 1992
92-54

STATE OF WASHINGTON

DEPARTMENT OF ECOLOGY

Mail Stop PV-11 • Olympia, Washington 98504-8711 • (206) 459-6000

August 1, 1992

TO: Interested Parties

FROM: Carol L. Fleskes, Program Manager
Toxics Cleanup Program

Toxics Cleanup Program

SUBJECT: Statistical Guidance for Ecology Site Managers

Attached is the August 1992 edition of Washington State Department of Ecology's (Ecology) "Statistical Guidance for Ecology Site Managers." The document provides guidance on statistical issues relating to the investigation and cleanup of soil- and groundwater contamination under the Model Toxics Control Act Cleanup Regulation. It is not intended for use at sites where routine petroleum leaking underground storage tank (LUST) cleanups are undertaken using Ecology's *Guidance for Remediation of Releases from Underground Storage Tanks*, which includes statistical guidance in an appendix.

Routine statistical procedures are provided in this Guidance that should be applicable to most sites. For statistical situations where site-specific decisions should be made, the Guidance provides Ecology staff with relevant information, but does not establish standard procedures or criteria. Consequently, some statistical methods and procedures are discussed that should not be used without site-specific approval of Ecology. Consult Section 1.2 (Using the Guidance Document) for more information. "Site-specific approval of Ecology" refers only to remedial actions conducted or ordered by Ecology, or to cleanups agreed to by Ecology in an agreed order or decree governing remedial actions at the site. Ecology will respond to questions relating to the Guidance from persons conducting independent cleanups if staff resources permit. However, it may be helpful to consult a statistician regarding sections of the Guidance that provide for site-specific decisions.

Important features of this Guidance include the default assumption of a lognormal distribution for soil and groundwater sampling data. This assumption was adopted on the recommendation of the Model Toxics Control Act Science Advisory Board. For data that do not follow a lognormal distribution, the Guidance provides statistical methods for rejecting the default assumption. Readers should also note that the Guidance provides new procedures relating to the use of background data in establishing a cleanup level. The technical basis for these procedures is explained in the document.

Ecology invites written comments from interested persons regarding this Guidance for consideration in making future revisions. Ecology's experiences in applying the Guidance to specific sites will also be considered in evaluating the need for revisions. More rapid updates will be provided through Guidance Supplements. These may be issued, for example, to cover a subject that is not presently addressed; to clarify a section that users find vague or ambiguous; or to replace a section in the current document.

Written comments on this document should be addressed to:

Nigel Blakley
Department of Ecology
Toxics Cleanup Program
P.O. Box 47600
Olympia, WA 98504-7600

Requests to be placed on the mailing list for Guidance Supplements and other Guidance mailings should be addressed to:

Sherrie Hanson
Department of Ecology
Toxics Cleanup Program
Statistical Guidance
P.O. Box 47600
Olympia, WA 98504-7600

CLF:NB:cp
Attachment

Washington State Department of Ecology

Statistical Guidance for Ecology Site Managers

Disclaimer

Notice: This document is intended solely for the guidance of Ecology staff. It is not intended, and cannot be relied on, to create rights, substantive or procedural, enforceable by any party in litigation with the State of Washington. Ecology reserves the right to act at variance with this Guidance at any time.

Toxics Cleanup Program
PO Box 47600
Olympia, WA 98504-7600

August 1992

Prepared by the Washington State Department of Ecology with assistance from PTI Environmental Services. Primary authors (in alphabetical order): Nigel Blakley, Greg Glass and Connie Travers.

CONTENTS

LIST OF FIGURES

LIST OF ACRONYMS

ANOVA	analysis of variance
ARAR	applicable or relevant and appropriate requirement
BDL	below detection limit
CI	confidence interval
EPA	U.S. Environmental Protection Agency
Ecology	Washington Department of Ecology
MLE	maximum likelihood estimator
MTCA	Model Toxics Control Act
PCBs	polychlorinated biphenyls
PLP	potentially liable person
PQL	practical quantitation limit
QA/QC	quality assurance/quality control
RCRA	Resource Conservation and Recovery Act
UCL	upper confidence limit
WAC	Washington Administrative Code

LIST OF NOTATIONS

μ	true mean of population
$\bar{x}$	mean of samples
n	number of samples
y_i	lognormally transformed sample data values
e	exponent (base e)
σ	true standard deviation of population
CV	coefficient of variation
x_i	sample data values
s	standard deviation of samples
X_p	percentile (e.g. X_{90} = 90th percentile)
P	percentile value
M	variable used for calculating 90th percentile of log-transformed data
d	denominator of W test statistic
a_i	coefficients calculated from Table A-1
α	significance level
k	tolerance interval coefficient
γ	significance level for tolerance interval
P_o	1- (percentile/100)
T_u	upper limit of tolerance interval
b_1, b_2	parameters for Conover (1980) nonparametric estimators of confidence intervals and percentiles for $n \leq 20$
$Z_{1-\alpha}$	percentile of normal distribution
v	variable used for calculating 10th percentile
u	upper rank of confidence limit
ln	natural logarithm
Σ	summation
b	tabled entries in Table A-5

1. INTRODUCTION

1.1 PURPOSE AND PHILOSOPHY OF THE STATISTICAL GUIDANCE DOCUMENT

This document is intended to provide Model Toxics Control Act (MTCA) site managers with guidance for sampling and analyzing groundwater and soils to develop background-based cleanup standards, where appropriate, and to determine whether a site or exposure unit meets cleanup standards. The cleanup standard may be listed in the regulation or established under applicable state and federal laws, or it may be set at natural background levels. Cleanup standards may also be established at calculated risk-equivalent concentrations.

We can never know the actual contaminant concentrations at a site unless we sample all the soil or groundwater present. Obviously, this is not feasible. However, we can draw conclusions about the site by sampling and statistically analyzing the results. We can estimate the parameters of the true contaminant concentration distribution based on the sample parameters. For example, we estimate the true average concentration at the site (μ) with the average of the samples ($\bar{x}$). This will always involve some uncertainty, because we can never be certain that the samples represent the true contaminant concentrations at the site. Suppose there is one small area of a site that is highly contaminated, but no samples are taken in that area. The conclusion, based on the samples, might be that the site is clean (uncontaminated). In this case, the samples are not representative of the true contaminant concentrations at a site because they do not reflect the highly contaminated area. Alternatively, suppose all the samples at the site were taken in the small, contaminated area. A conclusion might be reached that the site is very contaminated, when, in fact, only a small area is contaminated and most of the site is clean.

Two methods exist for handling the uncertainty in statistically representing contaminant concentrations at a site. One is to reduce the uncertainty by improving sampling design to include more samples—or more representative samples—because in general, the more samples that are collected, the more certain we can be that we are representing true site conditions. However, there will always be some uncertainty associated with the results. Alternatively, the uncertainty can be quantified by assigning **confidence intervals** (see Section 2.1.9) to the statistical parameters describing the samples [e.g., the **mean** (see Section 2.1.1)]. These intervals describe how confident we are that the true parameters lie within a range of values. For example, if the mean of a particular data set is 10, we could say we are 95 percent confident that the true mean of the data set lies between 3 and 16. This means that 5 percent of the time the true mean lies outside of this range of values.

Statistical methods presented in this manual are designed to permit site managers to make decisions about contamination levels at an entire site, or within an exposure unit, based on a limited number of samples. These methods are designed to take into account the uncertainty inherent in this process. MTCA provides for "other statistical methods" than those discussed in the rule. This document describes some other methods that may be applicable to a specific situation. *Other generally acceptable statistical methods exist for soil (EPA 1989a) and groundwater (EPA 1988).* References in the Bibliography provide additional statistical data evaluation methods (e.g., Gilbert 1987) which may be acceptable if consistent with MTCA requirements (e.g., see Section 2.1.6).

The philosophy behind the statistical procedures in MTCA includes the following principles:

1. Tests of compliance monitoring data should be such that a low frequency of relatively small-magnitude exceedances of the cleanup standard are allowable within the rules without triggering mandatory cleanup criteria, but that the frequency and magnitude of such exceedances should be limited.

2. In those cases where cleanup standards are based on background, the background distributions should be such that clean (i.e., uncontaminated) sites or exposure units have a high probability of being recognized as such.

An effort was made to make this document as applicable as possible to actual situations faced at sites. However, it is not possible to address every case that may occur in application to real-world situations. If a statistical interpretation of site data appears to be more complex than the examples provided in this document, it is recommended that the assistance of the Washington State Department of Ecology (Ecology) or a statistician be sought.

1.2 USING THE GUIDANCE DOCUMENT

Users other than Ecology staff:

Although this guidance should be used by all parties involved in the investigation and cleanup of hazardous waste sites under MTCA, the document was primarily written to assist Ecology staff. Thus while the guidance provides statistical procedures which may be used routinely at most sites, the document also provides information for Ecology staff on alternative approaches available under special circumstances (e.g. contaminant data are neither lognormally or normally distributed). **Decisions regarding the use of these alternatives are made by Ecology on a site-specific basis and therefore require consultation with the department.**

Information on alternative approaches requiring Ecology's approval is identified in this document in one of two ways. First, section headings are marked "*Requires consultation with Ecology*" where information is provided to Ecology staff for their use in making site-specific decisions. For example, the nonparametric method for estimating percentiles (Section 2.1.2.3) is only acceptable if Ecology has agreed to its use for a particular data set. Second, in other sections it is clearly indicated where consultation with Ecology is required before a specific statistical decision may be made (e.g. Section 4.3.5).

The requirement to consult with Ecology regarding sections of this guidance refers only to remedial actions conducted or ordered by Ecology, or to cleanups agreed to by Ecology in an agreed order or decree governing remedial actions at the site. The department will respond to questions relating to the guidance from persons conducting independent cleanups if staff resources permit. However, it may be helpful to consult a statistician regarding sections of the guidance which provide for site-specific decisions.

Overview for all users:

Basic statistical parameters and definitions, and methods for calculating these parameters, are described in Section 2. Section 2 should be read in its entirety by those unfamiliar with statistics, or it can be used as a reference and reminder for those more familiar with the material. However, guidance on distributions (2.1.4.2 - 2.1.4.3) is of key importance and should be read by all users of this document. Other important guidance also occurs at the end of this section (2.2 - 2.3). Section 3 describes issues to be considered in sampling. This is an extensive topic and will be addressed more fully in the future. Thus, this section is reserved in the current version of the guidance document. Section 4 describes the methods for answering the question, "What is the cleanup standard, and how is it related to background concentrations?" Both soils and groundwater are discussed. Section 5 describes the methodology for answering the question, "Does the site or exposure unit meet the identified cleanup standards?" Section 6 (Geostatistics) is reserved at this time. Section 7 contains general statistical references that provide additional information on topics covered in this guidance document. Numbered examples, mentioned throughout the text, are found in Section 8. Tables A-1 through A-7, along with other relevant material, are included in Appendix A.

Important terms are in bold face where they are introduced for the first time. If applicable, this will be followed by a reference to the section where this concept is discussed.

2. GENERAL STATISTICAL ISSUES

2.1 BASIC DEFINITIONS

The objective of this section is to describe basic statistical concepts and to act as a framework on which data interpretation and decisions may be based.

2.1.1 Mean

2.1.1.1 Arithmetic Mean (Example 1)—The arithmetic mean is the same as the average value of a data set. The mean value may not equal any of the data values. The mean, $\bar{x}$, may be calculated by summing the values in a data set and dividing by the total number of values in the set:

$$\bar{x} = \frac{(x_1 + x_2 + \ldots + x_n)}{n}$$

where

x_i = values of samples

n = number of samples.

The mean of the sampled values, $\bar{x}$, is likely to differ from the mean of the true **population** (see Section 2.1.4), μ, which could only be obtained by sampling all of the soils or groundwater at the site. Thus, we try to estimate the true mean, based on the sampled values. The mean of the sampled values may be influenced by **outlier** values (see Section 2.3) or by unrepresentative sampling of values within the distribution, which may give a biased view of the true overall statistical population. In the case of contaminant concentrations, samples below the detection limit must be handled carefully, so as not to bias the mean. Below-detection-limit data (known as **censored** data sets) are discussed in Section 2.2. In general, the arithmetic mean should be used for the statistical methods described in this document.

2.1.1.2 Geometric Mean (Example 2)—Environmental data are often analyzed using the geometric mean rather than the arithmetic mean, particularly for **lognormal** or other skewed data sets (discussed in Section 2.1.4.2). In this document, the mean is the arithmetic mean, unless it is specified otherwise. However, the geometric mean is mentioned here because it is often encountered in technical literature relating to lognormally distributed data.

The geometric mean is the nth root of the product of n numbers. For examples, the geometric mean of 6, 10, and 20 is the cube root of 6 x 10 x 20, or 10.63. In practice, the geometric mean may be estimated by the following method:

1. Transform the data by taking the natural logarithm (base e) of each value. Note that other transformations are acceptable (e.g., base 10 logarithms), but in this document the natural logarithm will be used. Most calculators have both logarithms, so care should be taken that the natural logarithm is used. Note that it is possible and acceptable to obtain negative values after transforming the data.

$$y_i = \ln x_i$$

2. Calculate the arithmetic mean of the transformed values:

$$\bar{y} = \frac{(y_1 + y_2 + \ldots + y_n)}{n}$$

where

y_i = lognormally transformed sample values

n = number of samples.

3. The sample geometric mean (for a base e logarithmic transformation only) is then:

$$e^{\bar{y}}$$

2.1.2 Median and Other Percentiles

Percentiles, also known as **quantiles**, describe a location in the distribution of data. For example, the 50th percentile is the value at which half the data lie above the value, and half lie below. For the 90th percentile value, 10 percent of the data lie above the value and 90 percent lie below. The 10th percentile is the point at which 90 percent of the data lie above the point, and 10 percent below.

2.1.2.1 Estimating the Median (Examples 3 and 4)—The **median**, like the mean, is a statistic that describes typical (central) values of the data set. The median is the 50th percentile of the data set: half the data values lie above the median and half below. As a measure of central tendency of the data set, the median is not influenced by extreme (very high or very low) values, as is the mean, but for this same reason, it also does not utilize all the information contained in the data set. The median can be estimated directly from the sample data using the following method:

1. Sort the data from smallest to largest, and rank them from 1 to n, where n is the total number of data points in the data set (sample size). If there is more than one data point with the same value (i.e., a "tie"), order the data points consecutively, and give each its own rank. For example, if the 5th and 6th lowest data points are both 28, assign one 28 a rank of 5 and the other a rank of 6. This will not affect the calculation of the median.

2. If the sample size, n, is odd: the sample median estimate is the (n+1)/2th value. For example, if the sample size is 19, the sample median is the (19+1)/2 = 10th value.

3. If the sample size, n, is even: the sample median estimate is the average of the n/2th and the (n+2)/2th values. For example, if the sample size is 20, the sample median estimate is the average of the 20/2 = 10th and the [(20+2)/2] = 11th values.

This method is illustrated in Example 3.

Alternatively, the median can be estimated from a **probability plot**. If the data are normally distributed (Section 2.1.4.1), plot the points on normal probability paper (included in Appendix A) and fit a line by eye to the points on a probability plot. Some statistical computer software packages can do this. Use the line to estimate the value corresponding to 50 percent on the cumulative percent scale. This value is the median. This method is demonstrated in Example 4. If the data are lognormally distributed, use a probability plot of the log-transformed data. Note that for the $\log_e$-transformed data, the value corresponding to 50 percent is the log of the median; you will have to convert it by taking the exponent (base e) of the transformed values. Alternatively, plot the points on **log-probability paper** and read off the median directly.

2.1.2.2 Estimating the 90th Percentile— Several methods are available for estimating the 90th percentile of a data set:

- If the data are lognormally distributed, calculate $\bar{x}$ and s for the $\log_e$-transformed data. Then calculate M, where $M = \bar{x} + (1.28)(s)$. The 90th percentile can then be approximated by:

$$X_{90} = e^{M}$$

(Note: the value of 1.28 is Z_{90}, which was obtained from Table A-6).

- If the data are normally distributed, the 90th percentile X_{90}, may be estimated from a probability plot. The procedure is basically the same as that for the median, but 90 percent on the cumulative percent scale is used. This method is recommended for censored data sets.

- If the data are normally distributed, calculate the sample mean ($\bar{x}$) and the sample standard deviation (s) (described in Section 2.1.3 below). The 90th percentile is approximated by:

$$X_{90} = \text{90th percentile} = \bar{x} + (1.28)(s)$$

This method is preferable for uncensored data sets. (Note: again, the value of 1.28 is Z_{90}, obtained from Table A-6).

2.1.2.3 General Nonparametric Method for Estimating the p^th Percentile (Example 5)—*[Requires consultation with Ecology.]*. If the data are neither normally nor lognormally distributed, a **nonparametric** method (Section 2.1.5), which does not require the data to fit any particular distribution, should be used. A normal or lognormal distribution should not be assumed if the statistical test indicates significant departure from either of these distributions. If a normal or lognormal distribution cannot be rejected, the best-fit distribution should be assumed, and the methods described in Section 2.1.2.2 should be used rather than a nonparametric method. A nonparametric (distribution free) method can be used to estimate any percentile, X_p, and is shown in Example 5.

1. Sort the data from smallest to largest, and rank them from 1 to n, where n is the total number of data points in the data set (sample size). Data points with the same value should be ordered consecutively, and each point assigned its own rank.

2. Estimate v

$$v=\frac{p}{100}(n+1)$$

   where v = the rank of the p^{th} percentile data.

3. If v is an integer, then the p^{th} percentile is simply the v^{th} ranked datum in the data set.

4. If v is not an integer, then the p^{th} percentile must be linearly interpolated between the two closest order statistics (see Example 5).

The nonparametric estimation of the median (50th percentile) value is seen to be a special case of this general method for estimating percentiles.

2.1.3 Standard Deviation, Variance (Example 6), and Coefficient of Variation

The standard deviation of the population, σ, represents the spread of the population around the mean. The standard deviation of the sampling data, s, which is an estimator of σ, can be calculated as the positive square root of the sample variance, s^2, which is defined by:

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$$

Calculation of s^2 and s is demonstrated in Example 6.

The coefficient of variation (CV), which is affected by the degree of skew (Section 2.1.4.2) is calculated as the standard deviation divided by the mean:

$$CV = s/\bar{x}$$

Most scientific calculators will calculate standard deviations. However, it is important to note whether the calculator divides by n or n−1 when performing the calculation. Some calculators will allow you to select the divisor. In general, the n−1 divisor should be used for calculating the standard deviation of a data set.

Note: do **not** use the standard deviation and mean of the sampling data when calculating the CV for compliance decisions (see Sections 4.3.3-4.3.5). Instead, use the standard deviation and mean of the best-fit distribution (Supplement S-5). For example, the CV of 3.65 calculated in Example 12 is for the best-fit lognormal distribution, not the raw data.

2.1.4 Probability Distribution

The probability distribution is a plot of the probability of a variable attaining a value. It is a curve, usually continuous, that shows all possible values and describes the true distribution of the population. In order to be valid, many statistical tests require that the data approximate a normal (or Gaussian) probability distribution (e.g., bell-shaped curve). For this document, a population can be thought of as the entire set of contaminant concentrations that could be measured at a site if all the soil or groundwater at the site could be sampled. Thus, it is not possible to know the true probability distribution of a population unless we sample all the soil or groundwater at a site, which, of course, is not feasible. Instead, we estimate the probability distribution based on only a sample of the population. The sampled data can then be plotted on a histogram. A histogram is a bar plot that shows ranges of discrete measured values, and the frequency with which these values occur in a data set. The probability distribution of the overall population can be inferred from the histogram (Figure 1).

2.1.4.1 Normal Distribution (Example 7)— A normally distributed population will form the familiar "bell-shaped," symmetric curve (Figure 2). Many statistical tests require that data be normally distributed. Several methods can be used to determine whether data follow a

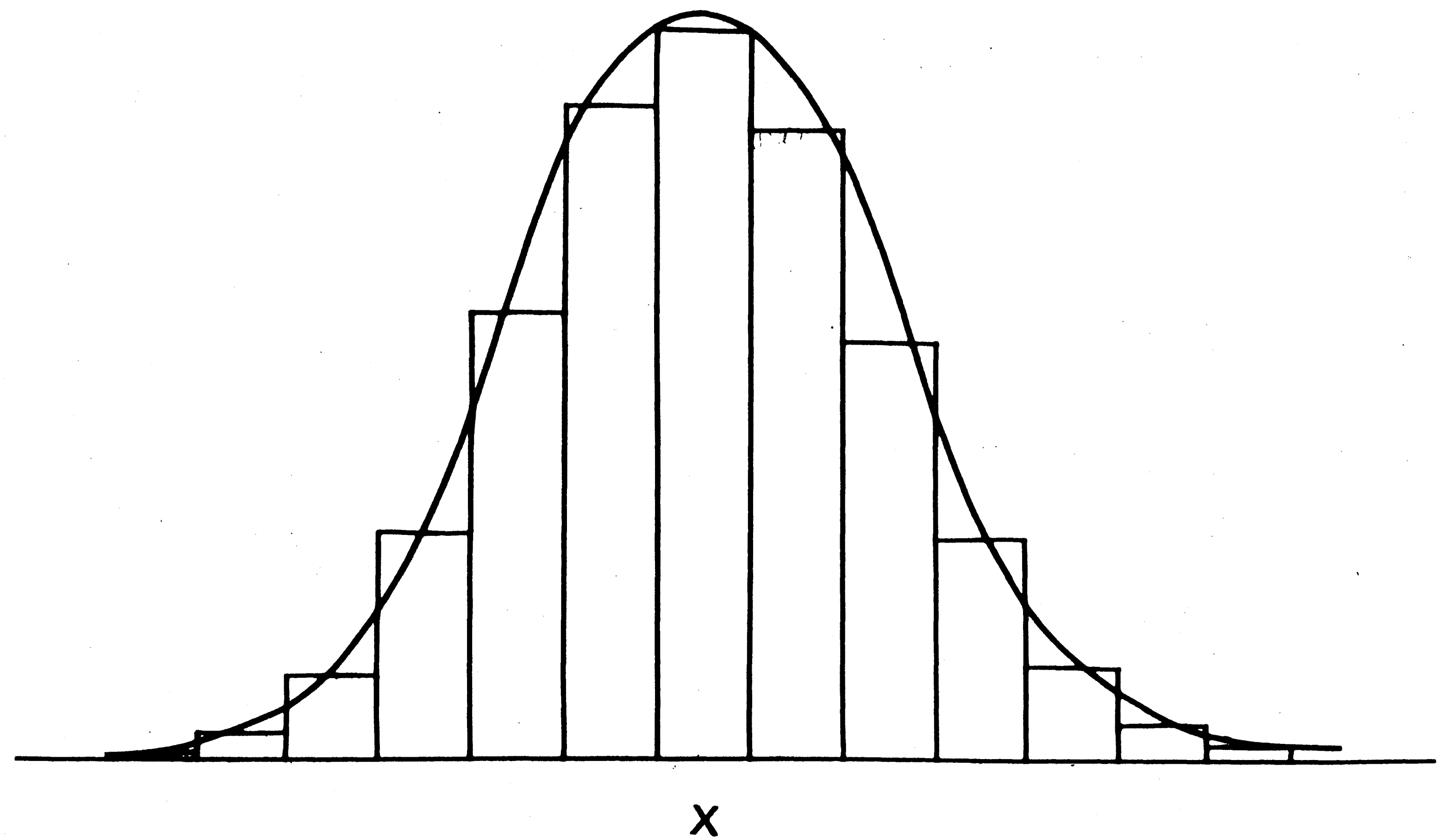


Figure 1. Relation between histogram and probability distribution.

C704-19-09 0392

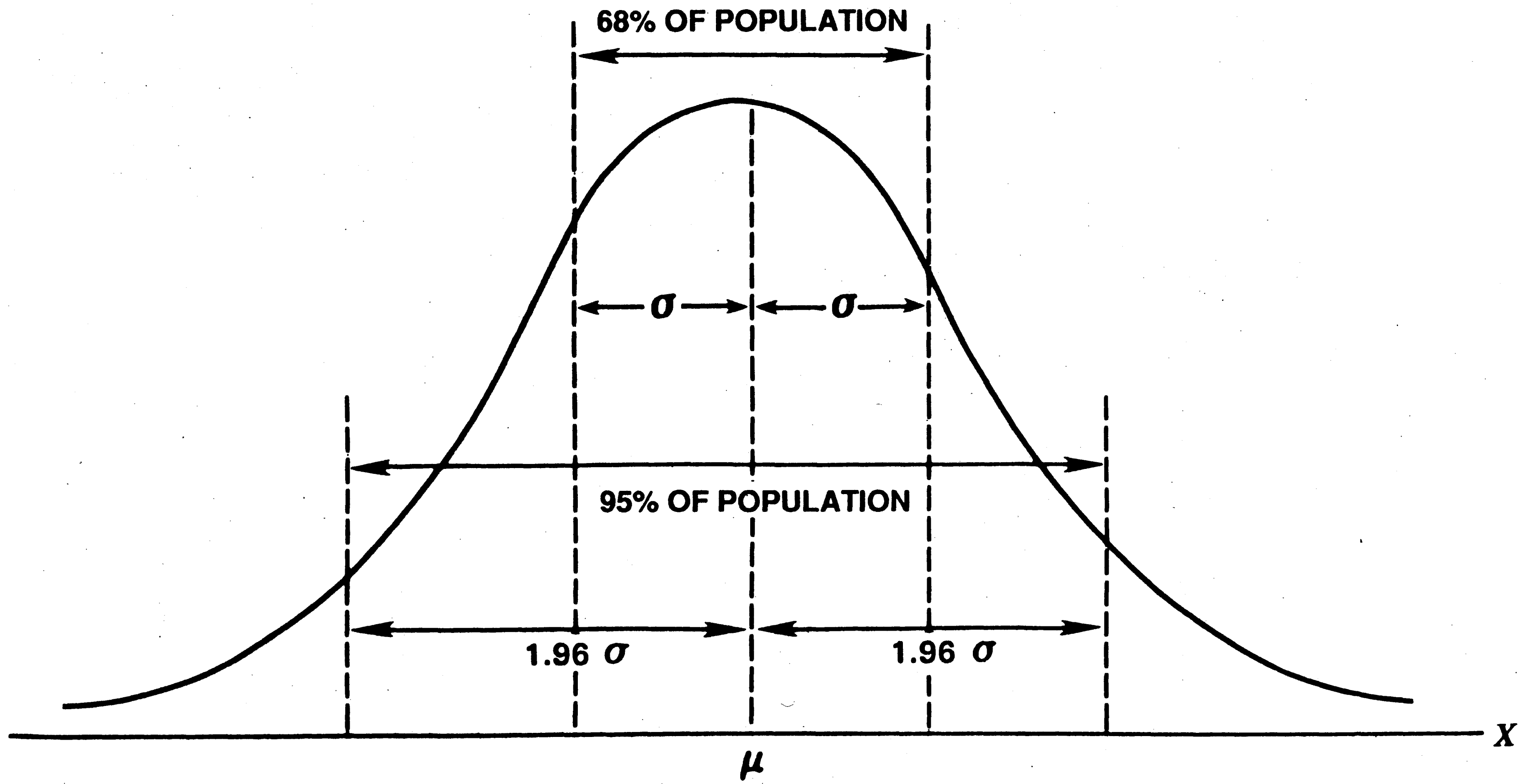


Figure 2. Normal distribution showing location of mean (μ) and standard deviation (σ) of underlying population.

C704-19-09 0392

normal distribution. The data for each contaminant at a site must be tested individually for normality.

Normality testing using probability plot—The simplest approach is to graph the data on a probability plot. Statistical computer software packages such as STATGRAPHICS® or SYSTAT® will do this for you; otherwise you will need probability plot graph paper if you do it by hand (linear [normal] probability plot paper is included in the appendix to this document). The measured data should be plotted on a normal probability plot. Then, a line should be overlaid that describes the data expected from a normal distribution with the same mean and variance as the measured data (Figure 3). The measured data points will not fall exactly on the line, but if they lie approximately on the line, the data are normally distributed. This is a somewhat subjective test. Several references are available that describe the development of normal probability plots (Neter and Wasserman 1974; Shapiro 1980).

Normality testing using the W test—The **W test** (Shapiro and Wilk 1965) can be used to test whether the data differ significantly from a normal distribution, but cannot be used to determine whether the data are normally distributed. If the W test does not show that the data differ from normal, a normal distribution can be assumed.

The W test, as described below, is appropriate for fewer than 50 samples. The W test is recommended by the U.S. Environmental Protection Agency (U.S. EPA 1986) because it performs well for small sample sizes (which are likely at MTCA sites). For larger sample sizes, D'Agostino's test should be used (D'Agostino 1971). Both tests are described in Gilbert (1987).

The W method tests the hypothesis: The data have been drawn from a normally distributed population. This is the **"null hypothesis"** for the test (the null hypothesis is discussed in Section 2.1.6). The alternative is that the underlying population is not normally distributed. The method for performing the W test is as follows (Gilbert 1987):

1. Compute the denominator, d, of the W test statistic. This is done by calculating the mean of the data set, $\bar{x}$, and subtracting the mean from each of the data values (some resulting values will be negative). The difference between the mean and each value should be squared, and the results should be summed. This is expressed by the following equation:

$$d = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

where

n = the total number of samples

x_i = the individual data values.

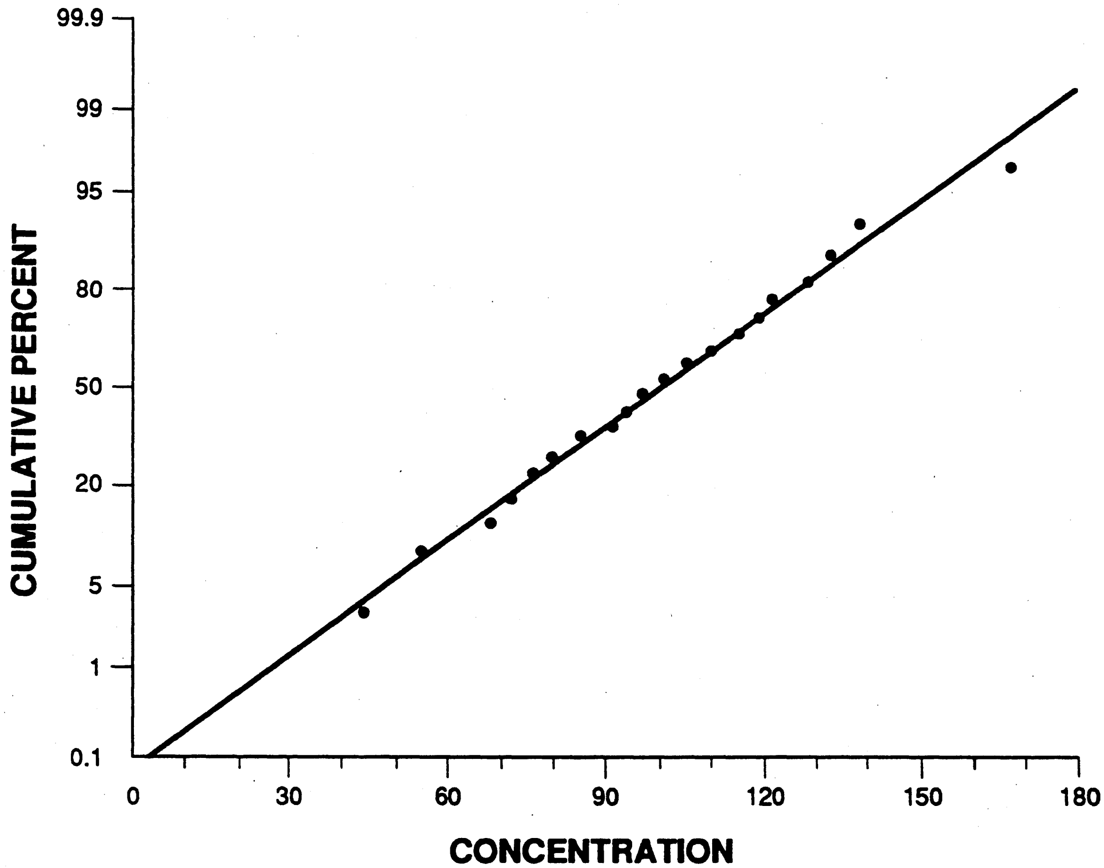


Figure 3. Data from Example 1 plotted on a probability plot. These data appear to be normally distributed.

C704-19-09 0392

2. Order the data from smallest to largest, and assign a rank to each value.

3. Compute r where

$$r = n/2 \quad \text{if n is even}$$

and

$$r = (n-1)/2 \quad \text{if n is odd.}$$

4. Use Table A-1 for the number of samples n to determine coefficients a_1, a_2, ... a_r.

5. Next compute W using the equation:

$$W = \left(\frac{1}{d}\right)\left[\sum_{i=1}^{r} a_i (x_{[n-i+1]} - x_{[i]})\right]^2$$

where

$x_{[i]}$ = the value of i^{th} ranked data

a_i = coefficients calculated from Table A-1.

6. Using Table A-2, find the value of W for a particular significance level, α, and sample size, n. A significance level of 0.05 (confidence level of 95 percent) is consistent with the significance level required by the regulations for other statistical tests. If the value for W calculated in Step 5 above is less than the value in Table A-2, the null hypothesis—that the population is normally distributed—should be rejected. If the W from Step 5 is greater than the tabled value for W, we can assume that the data are normally distributed. Example 7 demonstrates an application of the W test to the data in Example 4. Detailed instructions for examining data for departures from normality using the W test are given in Worksheet W-1a.

Normality testing by alternative methods—[*Requires consultation with Ecology.*]. Alternatively, the chi-square (χ^2) goodness-of-fit test at some specified significance level (e.g., 0.01) can be applied to test the normality of the data. The chi-square test is used to quantitatively evaluate the difference between the observed and expected frequency value for each variable. This test can be applied using computer software such as STATGRAPHICS®. Another available procedure, the nonparametric Kolmogorov-Smirnov test (Conover 1980), is considered to be more powerful than the chi-square test for evaluating the fit of a hypothesized distribution, particularly for small sample sizes (e.g., n<20). Several other methods for testing the normality of a data set are described in Shapiro (1980). Alternatives to the W test should not be used unless there is a valid statistical reason for doing so.

2.1.4.2 Lognormal Distribution (Example 8)—A probability distribution is symmetric if a vertical line can be drawn through the distribution such that the two sides are mirror images of each other (Figure 2). If a distribution is not symmetric with respect to the vertical line, it is skewed. Distributions may be skewed to the right or left. A distribution skewed to the left (also known as negatively skewed) will have a long tail on the left and a shorter tail on the right, while distributions skewed to the right (positively skewed) have a long tail on the right (Figure 4) and a greater proportion of the population on the left. Water quality data, and other environmental data, are often positively (sometimes highly) skewed (Gilliom and Helsel 1986; Gilbert 1987; Helsel 1990).

In this document, **the default assumption is that the data are lognormally distributed.** Data should first be tested to determine if a lognormal distribution is appropriate. If there is evidence that the data are normally distributed (e.g., visual fit or statistical test), or if the data do not appear to be lognormally distributed, they should be tested for normality. Rejection or acceptance of a lognormal or normal distribution can be made visually, but if there is any doubt, a statistical test should be performed to eliminate the subjectiveness of the visual methods. If both normal and lognormal distributions are rejected, the advice in the guidance document should be followed.

To test the assumption of lognormality, the data should be logarithmically transformed and tested for normality as described in Section 2.1.4.1. This involves calculating the natural logarithm (base e) of each of the data points. If the transformed data appear to be normally distributed (using the W test or D'Agostino's test) when they have been logarithmically transformed, the data set can be assumed to be lognormally distributed. Many of the statistical estimation methods and tests described in the following sections may then be performed on the transformed data. Detailed instructions for testing the assumption of lognormality using the W test are provided in Worksheet W-1. Supplement S-3 provides an overview of the procedure to follow in making a decision on the distribution of site or background data.

A histogram of a data set drawn from a lognormally distributed population is shown in Figure 5a. This data set was logarithmically transformed, and the transformed data appear to be normally distributed (Figure 5b). Logarithmic transformations are demonstrated in Example 8. In Figure 6, an untransformed data set is plotted on a probability plot, and the points do not plot on a straight line. However, the plotted, logarithmically transformed data approximate a straight line (Figure 7), indicating that the data set is approximately lognormally distributed (log [lognormal] probability plotting paper is included in the Appendix of this document). A comparison of normal and lognormal distributions is shown in Figure 8.

Lognormally transformed data should never be used to obtain summary statistics (e.g., mean, standard deviation) for the untransformed data, due to the transformation bias inherent in determining summary statistics for a transformed data set and then transforming the data back to original units. Thus, the mean of the log-transformed data is <u>not</u> the same as the logarithm of the mean of the raw (untransformed) data. However, transformation of percentiles (e.g., 90th percentile, median) does not exhibit this bias. In other words, the 90th percentile of the log-transformed data will be the same as the logarithm of the 90th percentile of the raw (untransformed) data.

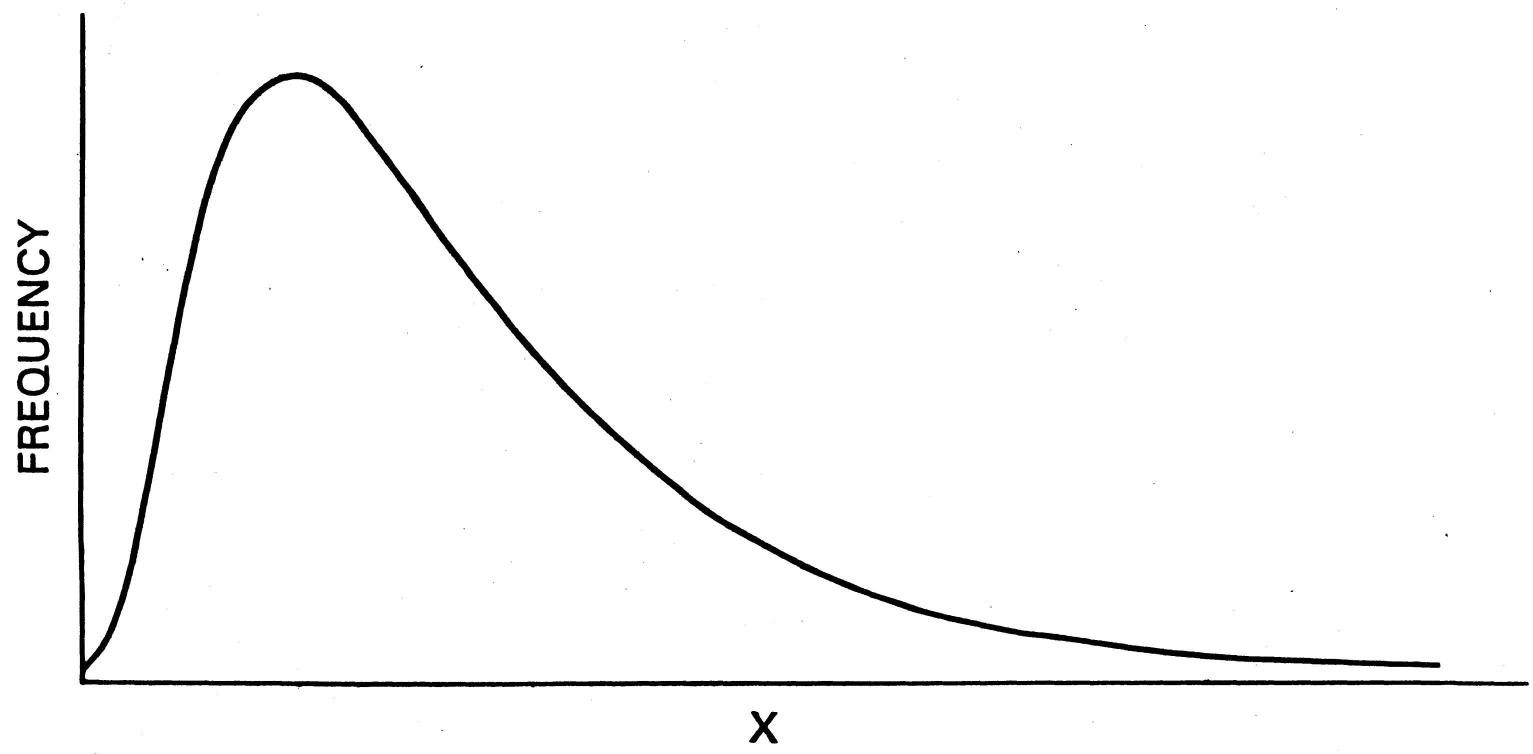


Figure 4. Positively skewed distribution.

C704-19-09 0392

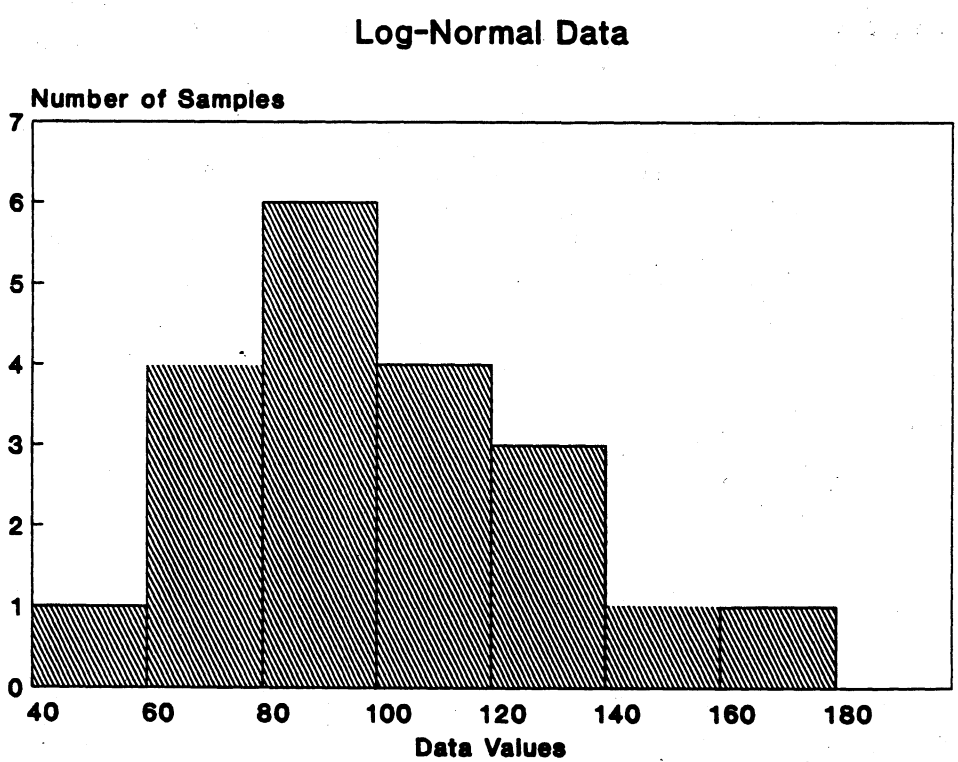


Figure 5a. Histogram of data from Example 8. These data appear to be lognormally distributed.

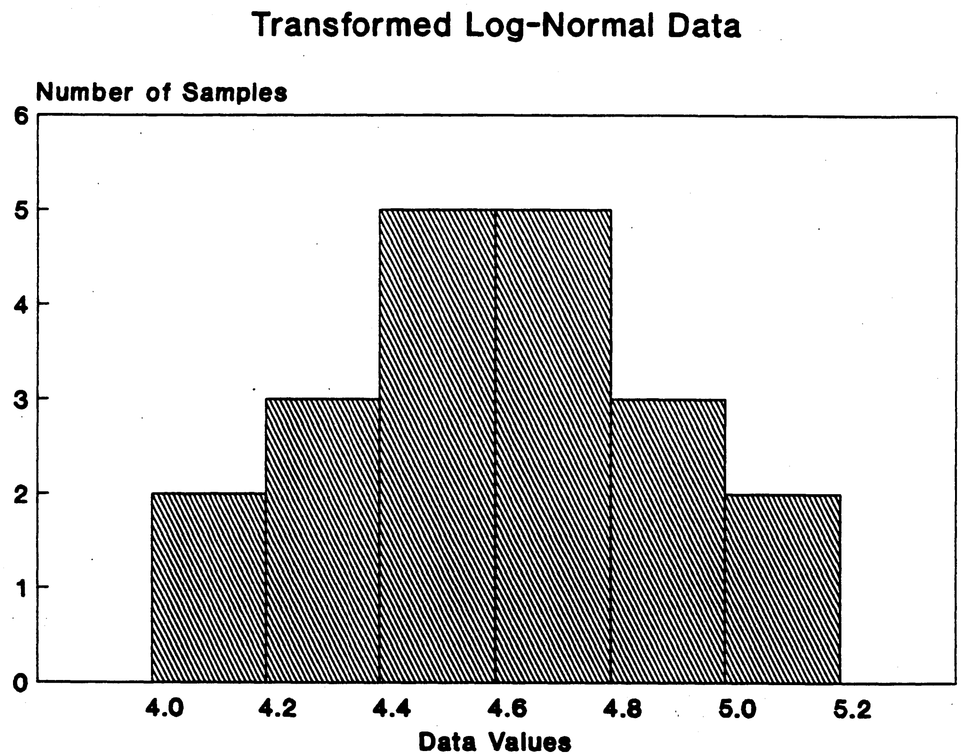


Figure 5b. Histogram of logarithmically ($\log_e$) transformed data from Figure 5a. These data appear to be normally distributed which suggests the original data are lognormal.

C704-19-09 0392

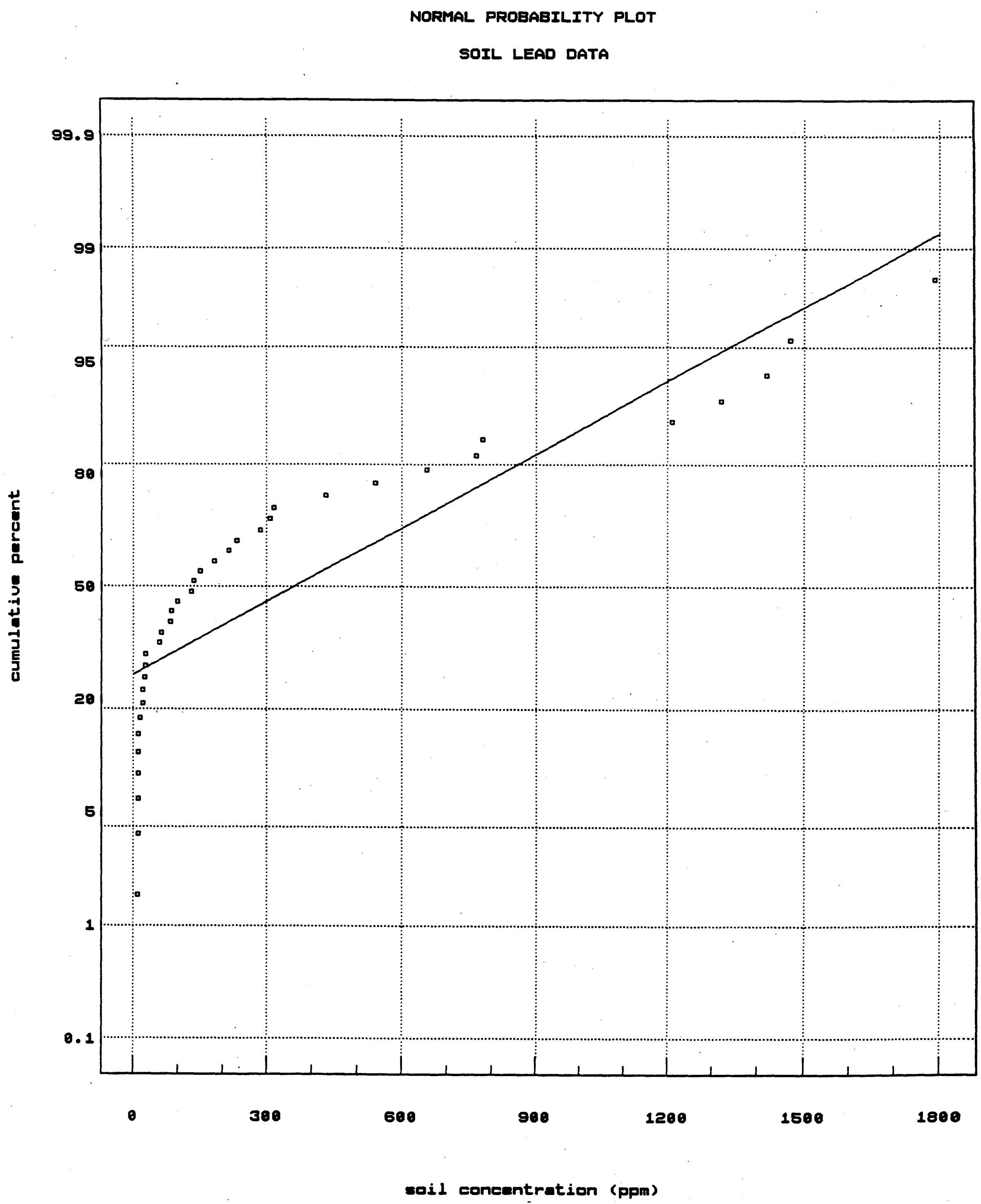


Figure 6. Soil lead data plotted on a probability plot. These data do not appear to be normally distributed (do not fall on the straight line).

C704-19-09 0392

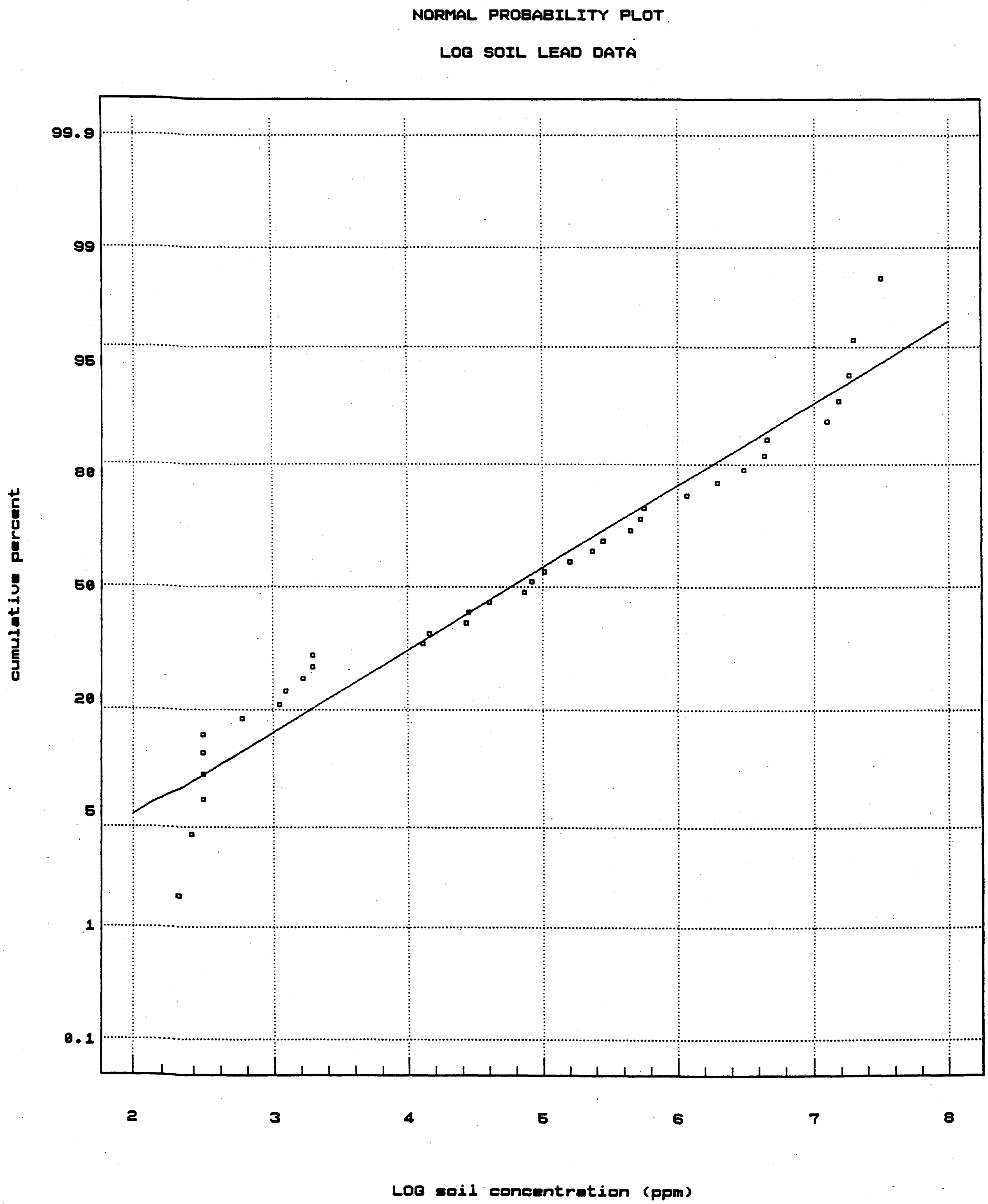


Figure 7. Log$_e$-transformed data from Figure 6 plotted on a probability plot. The log-transformed data appear to be normally distributed, indicating that the original data are lognormally distributed.

C704-19-09 0392

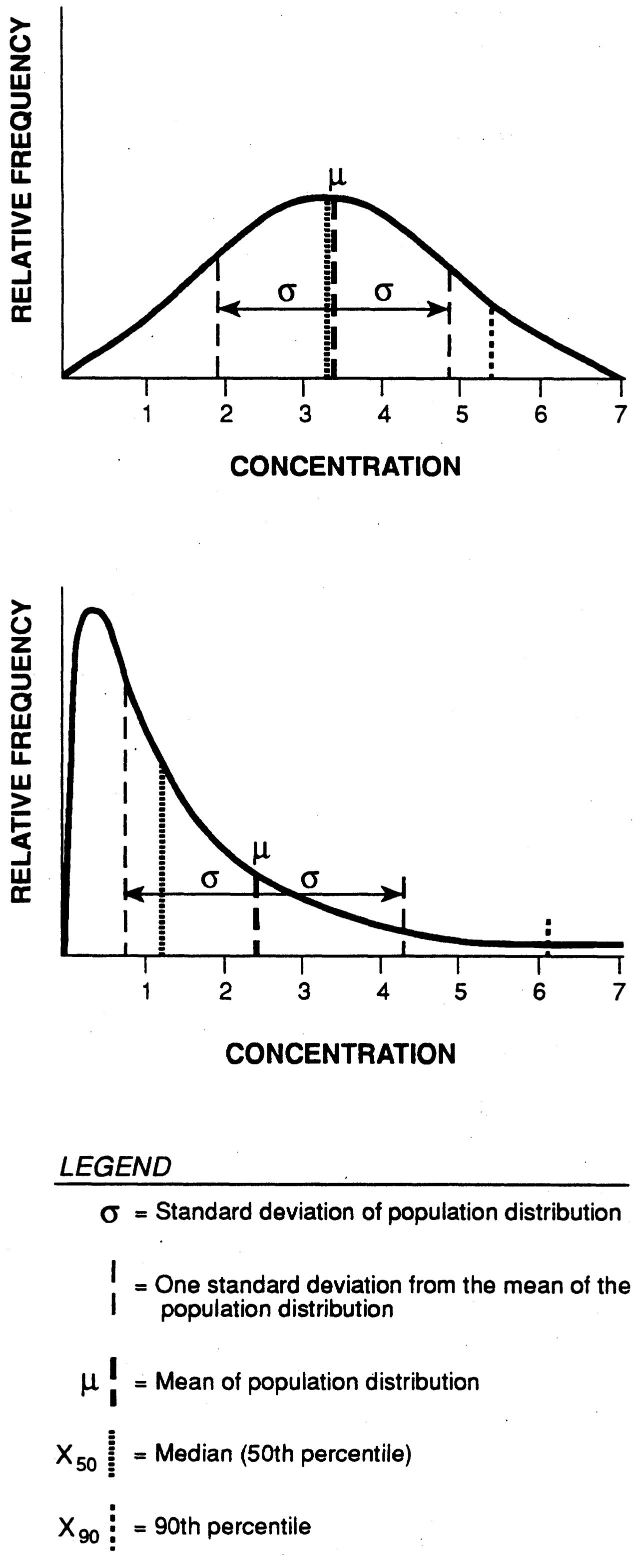


Figure 8. Relative position of mean, standard deviation and percentiles for a normally distributed population (upper figure) and a lognormally distributed population (lower figure).

2.1.4.3 Other Distributions—For small data sets (e.g., n < 20), it may not be possible to "reject" either the normal or lognormal distribution; both distributions may appear to fit the data. In this case, the lognormal distribution should be used. Alternatively, additional samples can be taken to better determine the distribution. This is demonstrated in Figure 9. Normal and lognormal distributions are sufficient to model many real-world statistical situations. However, some data sets may be neither normally nor lognormally distributed. Several other distributions have been used to model environmental data, including Weibull, gamma, and beta distributions. The three-parameter Weibull distribution can assume a wide variety of shapes and can be used to model both right and left-skewed data. These distributions are discussed briefly in Gilbert (1987), and are mentioned here because they may be encountered in statistical texts. However, for the statistical methods described in this document, if the data set does not appear to be normally or lognormally distributed, a nonparametric (distribution-free) statistical method should be used, if available and appropriate.

2.1.5 Parametric vs. Nonparametric Methods

Parametric estimation methods and tests require that the data be drawn from a population with a specific probability distribution (e.g., normal). When the distributional assumptions hold, parametric tests are usually more powerful than nonparametric (distribution-free) tests, although this is dependent on the type of test performed. However, parametric tests can lose statistical power or introduce bias if their distributional assumptions are incorrect. In this case, **statistical power** can be thought of as the ability of a method to detect site contamination if it is present, and to decide that remediation is unnecessary at a clean site. The loss of statistical power or introduction of bias when distributional assumptions are not met can render parametric statistical procedures ineffective in reaching decisions on site contamination.

Nonparametric estimation methods and tests, also called "distribution-free," do not require that the data be drawn from a specific distribution (e.g., normal). These methods and tests are valid for all data distributions. However, because parametric methods are generally more powerful if distributional assumptions hold, parametric methods are preferred unless data deviate significantly from normal and lognormal distributions. Thus, in order to use a nonparametric method, the distributional assumptions must be tested, and both the normal and lognormal distributions rejected.

2.1.6 Null Hypothesis

In MTCA (WAC 173-340-200), the null hypothesis (the "working assumption") is that contaminant concentrations at the site exceed the cleanup level (unless the cleanup level is based on background concentrations). The alternative is that they do not exceed the cleanup level. Since there is only one possibility for the alternative hypothesis, the appropriate statistical analysis is known as a **one-tailed test**. If there were two possibilities for alternative hypotheses, the test would be a **two-tailed test**.

The MTCA null hypothesis ("site exceeds cleanup level") is environmentally conservative but creates some statistical problems. This is because the conventional null hypothesis in

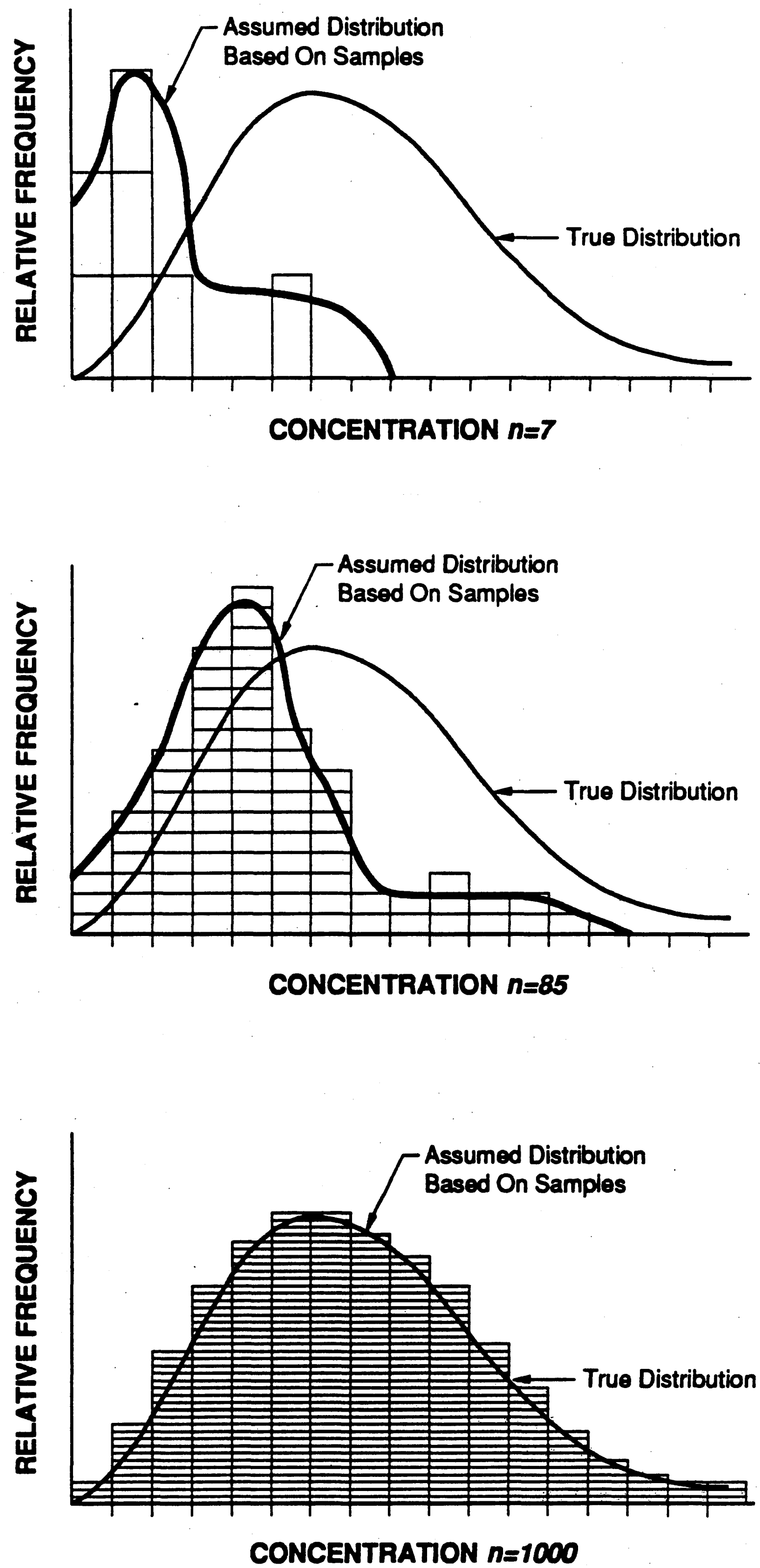


Figure 9. Larger sample sizes provide better estimates of the true population distribution.

C704-19-09 0392

statistics is the "no difference" hypothesis ("site is not different from cleanup level"). Most statistical tests are designed to test the "no difference" null hypothesis, and any introductory statistical textbook will be written from this perspective. Consequently, many commonly used statistical tests, such as the *t*-test or analysis of variance (ANOVA), are generally inappropriate for MTCA cleanups and are therefore not described in MTCA or this document. In addition, much of the information in statistics texts is also inappropriate. Statistical guidance published for the Resource Conservation and Recovery Act (RCRA) program is generally inapplicable to MTCA because the "no difference" null hypothesis is used in statistical analysis for RCRA facilities. However, U.S. EPA (1988, 1989a) has published statistical guidance for Superfund cleanups that is based on the same null hypothesis as MTCA, and may therefore be used for statistical analysis of data from MTCA sites. If in doubt, consult these sources or a statistician.

2.1.7 Type I and Type II Errors

Two types of errors can occur when a statistical test is applied to test a null hypothesis. If a statistical test shows that the null hypothesis is very unlikely, then we can accept the alternative, which in this case is that the site is clean. Since we are dealing with probabilities and not certainties in statistics, we could be wrong. If we are wrong—we assume that the site is clean and it is in fact contaminated—we have committed a Type I error. A Type I error means that the null hypothesis ("site exceeds cleanup level") is incorrectly rejected; a site that is actually contaminated will not be cleaned up. Statistics can't prevent Type I errors, but it does allow us to control the likelihood of committing such an error. In general, this likelihood is set at 0.05 (5 percent, or 1 time in 20) in the regulation [e.g., WAC 173-340-720(8)(e)(i)]. This defines what we mean by the null hypothesis being "very unlikely" and is an attempt to minimize mistakes. The statistical test must show that the chances of the null hypothesis being right are no greater than 0.05, or 1 in 20 in order to reject the null hypothesis.

The probabilistic nature of statistical decisions can also lead to a Type II error. When a Type II error occurs, the null hypothesis ("site exceeds cleanup levels") is incorrectly accepted. A statistical test on a particular data set may indicate that the null hypothesis is not sufficiently unlikely to justify its being rejected (i.e., it is more likely than 1 in 20), and the null hypothesis is therefore accepted. In this case, however, if the site actually is clean, we have committed a Type II error. If a Type II error occurs, cleanup will be required on a site that actually doesn't need it. In general, the likelihood of committing a Type II error can be reduced by collecting more samples or by using a more powerful statistical test. When deciding the number of samples needed at a site, it is worth considering that a Type II error may be a more expensive mistake than collecting too many samples. These issues are discussed further in the EPA guidance documents cited above.

2.1.8 Estimation Procedures vs. Statistical Tests

The procedures described in Sections 2.1.1–2.1.3 above are procedures for estimating summary statistics for the underlying population. Summary statistics include the mean, standard deviation, and median or other percentiles. Summary statistics describe basic facets of the data but do not provide interpretive or decision-making power.

In many cases, the purpose of statistical analysis is not only to estimate the statistical parameters for the underlying population, but also to make some conclusion about those data. Statistical tests have been developed for this purpose. In its simplest form, a statistical test deals with hypotheses and estimating the likelihood that they are correct. The following sections describe methods to reach conclusions about the site contaminant concentration data.

The concepts behind a statistical test of a null hypothesis will be illustrated by an example:

> Samples are collected from two orchard fields to measure soil arsenic levels. There are two hypotheses. One is that the soil arsenic level is the same in both fields. The alternative hypothesis is that the soil arsenic levels are different. Normally in statistics the first hypothesis ("no difference") is the null hypothesis. A statistical test can then be used to estimate the likelihood that the null hypothesis is correct. If it is "very unlikely," then the alternative, that the orchard fields are different, is probably correct. This example illustrates several important points. First, we can only test the null hypothesis, we cannot prove it. Second, statistics doesn't provide certainty (although it does let you specify what you mean by "very unlikely"). A more subtle point is that if the null hypothesis is probably wrong, and there is a difference between the fields, then there are actually two possibilities: field #1 has higher arsenic levels than field #2, or vice versa (either way, the fields are different). Where the alternative to the null hypothesis contains two possibilities, the appropriate analysis is a two-tailed statistical test.
>
> If field #1 had been sprayed with an arsenic pesticide, there is good reason to set up different hypotheses. Now the alternative hypothesis could be that the soil in field #1 has more arsenic than the soil in field #2, and the null hypothesis is that it doesn't. If a statistical test shows that the null hypothesis is very unlikely, there is only one possibility left: arsenic levels in field #1 are higher than in field #2. Here the appropriate analysis is a one-tailed statistical test.

In general, estimation methods are not influenced by the null hypothesis, whereas statistical tests are. The procedures discussed in this document are estimation procedures, and therefore are not influenced by the null hypothesis to be tested (the null hypothesis does not influence the confidence interval or tolerance interval "tests" described in this document). However, "alternate statistical procedures" are allowed by MTCA. If methods other than those described in the regulations are used, they must be consistent with the MTCA null hypothesis that the site exceeds the cleanup level. As mentioned above, many of the common statistical tests (e.g., *t*-test) are not appropriate for this null hypothesis.

2.1.9 Confidence Interval

Estimation procedures do not provide population parameters (e.g., mean) with absolute certainty. The confidence interval for statistical parameters can be used to describe the likelihood that the parameters will fall within a specific interval.

Suppose that a specific number of samples are taken at a site, the $100(1-\alpha)$ percent confidence interval is calculated, and this process is repeated many times. The $100(1-\alpha)$ percent confidence interval (CI) on a percentile (e.g., the median) means that $100(1-\alpha)$ percent of those intervals will include the percentile. The level of significance, α, is calculated from the selected CI by the following equation:

$$\alpha = 1 - CI/100.$$

Thus, for a 95-percent confidence interval, α is 0.05. For a significance level $\alpha = 0.05$, the 95-percent CI on the median means that the true population median will be within the interval 95 percent of the time.

2.1.10 Tolerance Interval

A **tolerance interval** is based on determining the confidence interval on a fixed proportion of the measurements, rather than on a particular parameter (e.g., the median). A confidence interval describes the likelihood that the particular parameter (e.g., the median) will fall within the interval. The tolerance interval describes the likelihood that a portion of the measurements (e.g., 95 percent) will fall within a specific interval. For example, the value obtained from the upper 95-percent tolerance interval around the 90th percentile means that we are 95 percent confident that at least 90 percent of the distribution is less than the value.

The tolerance limits are given by

$$\bar{x} \pm ks$$

The k value is essentially a factor that reflects the percentile of interest and the sample size. It increases the standard deviation by an amount related to the number of samples and the confidence level desired.

The tolerance interval approach assumes that the sampled data are drawn from a normally distributed population. This approach is more sensitive to the normality assumption than the confidence interval approach. It should not be used for data where a statistical test indicates that the normal distribution is inappropriate. For lognormally distributed data, see Section 5.2.2.2. Methods for data that are neither normally or lognormally distributed are described in Sections 5.2.2.3 and 5.2.2.4.

2.2 SAMPLES WITH VALUES BELOW THE DETECTION LIMIT OR PRACTICAL QUANTITATION LIMIT

Environmental data sets commonly contain data that are reported as "less than" the detection limit, or "not detected." This is particularly common for contaminants such as volatile organics, which are not normally present in the environment. In addition, due to conditions such as matrix interference, a laboratory measurement may be above the method detection limit, but below the practical quantitation limit (PQL), and these measurements will commonly be reported as "less than" the PQL. Data sets that contain below-detection-limit (BDL) or below-PQL data are known as **censored** data sets. Censored data sets present difficulties for many standard estimation procedures and statistical tests. For example, the mean cannot be estimated by the method described in Section 2.1.1 unless numerical values are assigned to the BDL or below-PQL data. Thus, the values assigned to BDL and below–PQL data could have a significant impact on the calculated mean for the data set. Censored data are less influential, however, when we are interested in upper-percentile estimates (e.g., defining background concentrations).

The method described in MTCA for handling censored data sets is the same as that used for estimating background concentrations, and for demonstrating compliance with groundwater, surface water, and soil cleanup levels. The regulation requires that all concentrations below the detection limit be assigned a value equal to one-half the detection limit of the method being used. Measurements above the method detection limit, but below the PQL shall be assigned a value equal to the method detection limit [WAC 173-340-708(11)(e), 173-340-720(8)(g), 173-340-730(7)(f), 173-340-740(7)(g)]. However, "alternate statistical procedures" for handling censored data may be approved by the department.

2.2.1 Additional Information

Three basic methods are available for estimating summary statistics for censored data sets: 1) simple substitution, 2) distributional methods, and 3) robust methods (Helsel 1990). These methods range from simple to complex. The method described in MTCA is an example of simple substitution, which involves substituting a single value for each BDL or below-PQL value. Many studies have found that simple substitution methods do not estimate summary statistics of the underlying population as well as more complicated methods for handling censored data (Helsel 1990). [*Use of methods 2) or 3) requires consultation with Ecology.*]

Distributional methods estimate a distribution for the data and use the characteristics of the distribution to estimate summary statistics. Helsel (1990) states that the best estimation method in this category is the maximum likelihood estimator (MLE). MLEs have performed well for percentile estimation, but not as well for estimating the mean and standard deviation of a data set. This method is accurate only if the data fit the assumed distribution well, and the sample sizes are large (e.g., >30) (Helsel 1990). Due to the small sample sizes likely to be available at MTCA sites, however, these methods may not be appropriate.

Helsel (1990) recommends the use of robust methods for estimating the mean and standard deviation. These methods use the observed data above the detection limit to assume a distribution, and then extrapolate the distribution below the detection limit to calculate summary statistics. If the data above the detection limit fit a normal or lognormal distribution, this can be done with a probability plot. Robust methods are recommended when data do not appear to fit the assumed distribution well.

The percentage of data below the detection limit will influence which methods are applicable for a particular data set. If the data set contains only a small percentage of censored data (e.g., no more than 15 percent), simple substitution methods will be satisfactory for estimating parameters. However, for estimation of summary statistics such as the mean and standard deviation, the presence of a substantial number of BDL or below-PQL data poses a significant problem from a statistical standpoint, unless more robust methods—discussed above—are used. However, some statistical estimation methods are not influenced by censored data. For example, the nonparametric confidence intervals about upper percentiles (Sections 5.2.2.3 and 5.2.2.4) will not be influenced by some censored data.

2.2.2 Multiple Detection Limits [*Alternative methods require consultation with Ecology.*]

Data sets may contain data with more than one detection limit. This may occur when data sets from multiple laboratories are combined, or data are analyzed at different times with variations in the reporting limit (usually the limit becomes lower over time). Using the simple substitution method (one-half the detection limit) described in the MTCA, multiple detection limits will not pose a problem. However, if alternative methods for handling BDL data are used, multiple reporting limits may cause some difficulties. Helsel (1990) recommends using robust methods for estimating the mean and standard deviation, and MLEs for percentiles.

2.3 OUTLIERS

The EPA groundwater guidance (U.S. EPA 1988) states:

In many statistical texts, measurements that are very large or small relative to the rest of the data, or are suspected of being unrepresentative of the true concentration at the sample location are often called ***"outliers."*** *Observations which appear to be unusual may correctly represent unusual concentrations in the field, or may result from unrecognized handling problems, such as contamination, lab measurement, or data recording errors. If a particular observation is suspected to be in error, the error should be identified and corrected, and the corrected value used in the analysis. If no such verification is possible, a statistician should be consulted to provide modifications to the statistical analysis that account for the suspected "outlier" ... The handling of outliers is a controversial topic. In this document, all data not known to be in error are considered valid because:*

- *The expected distribution of concentration values may be skewed (i.e., non-symmetric) so that large concentrations that look like "outliers" to some analysts may be legitimate;*
- *The procedures recommended in this document are less sensitive to extremely low concentrations than to extremely high concentrations; and*
- *High concentrations are of particular concern for their potential health and environmental impact.*

There are no provisions in MTCA for excluding "outliers" that cannot be demonstrated to be in error.

3. SAMPLING

A wide variety exists in sampling designs. Each describes the number of samples, locations for sampling, type of samples, and time frame for sampling. The sampling plan should be considered carefully prior to performing any sampling or performing statistical analysis on data, because a poorly designed sampling plan can greatly reduce the usefulness of the collected data. The sampling method used can influence the effectiveness of the remedial action in protecting human health and the environment. Sampling should be continued until the complete, preplanned sampling workplan has been carried out. It is unacceptable to terminate a sampling plan prematurely because the data collected to date indicate the results desired by the sampler (e.g., the cleanup level has been met).

Many factors should be considered in sampling, including the objectives of the study, the sampling method (e.g., random vs. systematic sampling), cost effectiveness of the sampling program, statistical analysis to be performed on the data, and the expected type and distribution of contaminants. In addition, several practical factors exist, such as legal and political considerations (e.g., sampling on private property), site accessibility and availability, and required equipment, which may affect sampling design. These factors influence sampling locations as well as the number and type of samples required. Sampling design is an extensive topic, and is beyond the scope of this document. However, its importance should not be underestimated. In this document, it is assumed that sampling design issues have been considered prior to performing statistical analyses of data.

Gilbert (1987) presents concepts and considerations for several sampling methods (e.g., simple random, stratified random, systematic). Soil sampling locations are discussed in McBratney et al. (1981); McBratney and Webster (1981) and U.S. EPA (1989a). Design of groundwater monitoring systems is described in Nelson and Ward (1981) and Sophocleous et al. (1982).

Special Comment on Hot Spots: No discussion of "hot spots" (highly contaminated local areas) is presented in MTCA. Gilbert (1987) presents a method for locating single hot spots by sampling on a square, rectangular, or triangular systematic sampling grid. Methods for locating multiple hot spots are presented in Gilbert (1982) and Holoway et al. (1981).

Special Comment on Compositing: Compositing of soil, or occasionally groundwater, samples refers to taking several samples and combining them into a single sample for analysis. This is commonly done to reduce analytical costs. There are two common methods used for compositing samples. The first method entails sampling segments of the soil core at random or at systematic locations. The sampled portions are homogenized and then subsampled. The second method requires retaining the entire soil core, homogenizing all the material, and then subsampling. The second method is preferable from a statistical standpoint, because the subsampling variance will be lower (U.S. EPA 1989a).

Compositing may be useful in screening a large area for contamination (e.g., screening for hot spots) in a cost-effective manner. In addition, compositing has been used successfully to evaluate the risk associated with an "exposure unit," the area over which people are expected to be exposed at a site and where cleanup actions are being considered (Ryti and Neptune 1991). In this case, the average concentration of contaminants over an exposure unit is a meaningful basis for assessing risk, and thus, compositing is a useful sampling technique (Neptune et al. 1990).

Despite the advantages associated with compositing, there are several problems that should be considered prior to sampling.

- A contaminated sample may be overlooked due to the effects of dilution. For example, suppose the detection limit for a particular contaminant is 1 mg/kg, and the action level is 3 mg/kg. Ten samples are taken and composited into one sample. If one sample has a concentration of 9 mg/kg, and all of the other samples are uncontaminated, the dilution effect of mixing the single contaminated sample with all the clean soil will cause the overall concentration measured in the soil to be below the detection limit of 1 mg/kg, and the soil will be considered clean. However, the local, hotspot concentration of 9 mg/kg is greater than the 3 mg/kg action level, and the site actually should be considered contaminated.

- Compositing methods may be inappropriate unless the statistical parameter of interest is the mean concentration. This is because the variance of the mean contributed by differences in location across the site from composited samples will be lower than the same variance associated with the mean from noncomposited samples (U.S. EPA 1989a).

- For contaminants such as volatile organics, compositing may cause the loss of material from the soil sample, and will thus reduce the measured contaminant concentration.

Due to these problems, compositing should be used only when it is supported by defined sampling objectives and its use can be shown to be appropriate for those objectives. Unless there is a well-defined reason for compositing, it should not be performed.

Several references are available that describe compositing of soil samples, including Duncan (1962), Rohde (1976), Schaeffer and Janardan (1978), Elder et al. (1980), U.S. EPA (1983, 1984, 1989b), Neptune et al. (1990), and Ryti and Neptune (1991).

Special Comment on Variability and Error in Data: Variability in environmental data can be attributed to two primary factors: 1) true variability in the population and 2) analytical or statistical uncertainty or error. True variability in contaminant concentrations in soils and groundwater may be due to a wide variety of factors, including:

- Natural variations in the geologic media (e.g., composition, permeability, and grain size)

- Distance from the source of contamination and variations in the source over space and time

- Differences in vegetation and in activity of microorganisms
- Temporal and spatial variations in background levels
- Chemical reactions of contaminants (e.g., degradation and transformation)
- Seasonal variation (e.g., in precipitation or temperature).

In addition, several sources of error and uncertainty exist that can result in observed variability in sampled data:

- Measurement bias (constant factor by which measurements are too high or low)
- Uncertainty in measurements (random sampling error)
- Quality assurance and quality control (QA/QC) problems. It is critical that the data available at the time of statistical evaluations have been through a QA/QC or data validation step and that they are deemed useful as reported for further decision-making at the site.

4. DETERMINATION OF CLEANUP STANDARDS AND BACKGROUND CONCENTRATIONS

4.1 DECISION-MAKING PROCESS

This chapter addresses three issues:

1. In general, how are cleanup standards determined?
2. What are the criteria for using background concentrations to determine a cleanup level?
3. How should the background data be used to set a cleanup level?

Prior to evaluating onsite data, the cleanup standard should be determined for the contaminants present at the site. This standard may be based on appropriate applicable state and federal laws, risk, ecological factors, and analytical considerations (e.g., BDL data, PQL), or may be related to background levels of the contaminant near the site. The process involved in choosing a method for determining a cleanup level is shown in Figure 10.

4.2 WASHINGTON ADMINISTRATIVE CODE DEFINITIONS

4.2.1 Establishing Cleanup Levels: Methods A, B, and C

Establishing cleanup standards (WAC 173-340-610) requires the specification of:

1. Cleanup levels
2. Points of compliance (locations where cleanup levels must be met) and time of compliance (for groundwater)
3. Additional regulatory requirements that apply to a cleanup action because of the type of action and/or the location of the site.

MTCA provides three basic methods for establishing cleanup levels in groundwater, surface water, soil, and air (WAC 173-340-700). Cleanup levels resulting from these methods may be broadly defined as:

- Method A—numerical standards (routine cleanup method)
- Method B—site-specific method that includes risk-assessment-based standards, standards based on applicable state and federal laws, or background concentrations

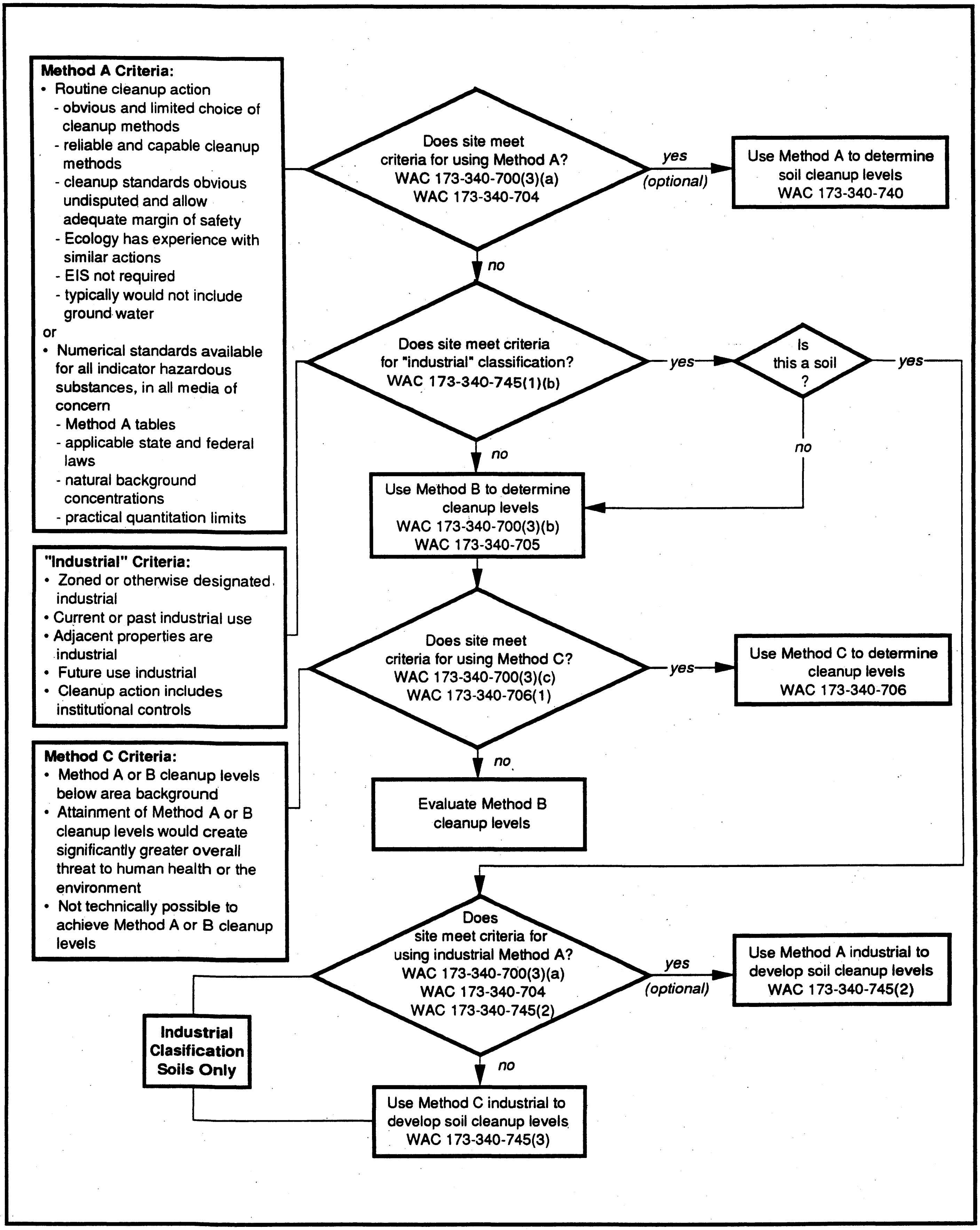


Figure 10. Flowchart for determining whether Method A,B, or C should be used for establishing cleanup levels.

(standard method)

- Method C—when compliance with Method A or B cleanup levels is impossible or may cause greater environmental harm or if site is an industrial site (conditional method).

4.2.1.1 Method A: Tables—Method A can be applied if either of the following conditions are met (WAC 173-340-704):

1. The site qualifies for a routine cleanup action. A cleanup action can be considered "routine" if all of the following criteria are met [WAC 173-340-130(7)]:

   - It involves an obvious and limited choice of cleanup methods
   - It uses a cleanup method that is reliable and has been proven capable of achieving cleanup standards
   - Cleanup standards for each hazardous substance addressed by the cleanup are obvious and undisputed, and allow an adequate margin of safety for protection of human health and the environment
   - Ecology has experience with similar actions
   - An environmental impact statement is not required.

   Cleanup of groundwater will not normally be considered a routine cleanup action [WAC 173-340-130(7)(c)].

2. Numerical standards are available for all indicator hazardous substances in all media of concern. Numerical standards may be available in the regulations (Tables 1, 2, and 3 of WAC), or applicable state and federal laws. Under Method A, cleanup levels must be at least as stringent as concentrations specified in these sources. If they are not available from these sources, cleanup standards can be set at natural background concentrations or the PQL for the substance in question. Ecology may set more stringent standards if needed to protect human health and the environment.

4.2.1.2 Method B: Standard Method—The regulations [WAC 173-340-700(3)(b)] state that under Method B:

...cleanup levels for individual hazardous substances are established using applicable state and federal laws or the risk equations specified in WAC 173-340-720 through 173-340-750. For carcinogenic compounds, cleanup levels are based upon the upper bound of the estimated excess lifetime cancer risk of one in one million. For individual noncarcinogenic substances, cleanup levels are set at concentrations which are anticipated to result in no acute or chronic toxic effects on human health and the environment. Where a hazardous waste site involves multiple hazardous substances and/or multiple pathways of exposure, Method B cleanup levels for individual substances must be modified in accordance with the procedures in WAC 173-340-708. Under this method, the total excess lifetime cancer risk for a site shall not exceed one in one hundred thousand and the hazard index for substances with similar noncarcinogenic toxic effects shall not exceed one (1).

4.2.1.3 Method C: Conditional Method—Method C cleanup levels may be established based on applicable state and federal laws and a site-specific risk assessment if any of the following conditions are met (WAC 173-340-706):

1. Cleanup levels established using Method A or B are below area background concentrations.
2. Attainment of Method A or Method B cleanup levels has the potential for creating a significantly greater overall threat to human health or the environment than attainment of Method C cleanup levels.
3. Method A or Method B cleanup levels are below technically possible concentrations. "Technically possible" means that remedial measures are capable of being designed, constructed, and implemented in a reliable and effective manner, regardless of cost (WAC 173-340-200).
4. The site is defined as an industrial site (see WAC 173-340-745) and meets the criteria for establishing soil cleanup levels under WAC 173-340-745:
   - The site is zoned for industrial use
   - The site is currently used for industrial purposes
   - Adjacent properties are currently used for industrial purposes
   - The site is expected to be used for industrial purposes in the foreseeable future
   - Institutional controls will be implemented as part of the remedial action.

Additional criteria for using Method C include:

- All ARARs will be met
- All practicable methods of treatment will be used
- Institutional controls will be implemented

A flowchart for use in determining whether Method A, B, or C is appropriate for establishing cleanup levels at a site is shown in Figure 10.

4.2.2 Natural vs. Area Background

The MTCA regulation makes a distinction between natural and area background concentrations.

4.2.2.1 Natural Background—Natural background refers to the concentration of a constituent that occurs naturally in the environment and has not been influenced by localized human activities. An example presented in MTCA (WAC 173-340-200) is that several metals occur naturally in the bedrock and soils of Washington State due solely to the geologic processes that formed these materials; therefore, the concentrations of these metals would be considered natural background. In addition, some constituents have been used globally, and low concentrations of these contaminants can be found in soils and groundwater throughout much of the state. These concentrations are the result of widespread use of the constituents and not localized human activity. Examples presented of constituents for which low concentrations would be considered natural background include polychlorinated biphenyls (PCBs) and radionuclides (due to fallout from bomb testing and nuclear accidents).

For comparison of onsite constituent concentrations with natural background levels, data should be obtained from a suitable reference area that is comparable to the site (e.g., similar geology and soil characteristics).

4.2.2.2 Area Background—Area background is defined as the concentration of hazardous substances that are consistently present in the environment in the vicinity of a site, and are the result of human activities unrelated to releases from that site. The size of the area affected by a particular contaminant is smaller for area background levels than for natural background. For example, lead levels in Seattle might be higher than lead levels in Bellevue; area background concentrations would therefore be different in these two cities.

4.3 SOIL CLEANUP STANDARDS BASED ON BACKGROUND DATA

MTCA regulations allow background concentrations to be considered in establishing cleanup standards. The role of background concentrations within the regulation and the procedures for background data evaluations are discussed in the sections below. Details of the statistical methods used for background data evaluations are provided in Sections 2 and 5.

4.3.1 Characteristics of Background Data Sets

Several characteristics of background data should be recognized:

- Background data are variable, and samples will typically reflect a range of values, not a single value. Therefore it is appropriate to consider the distribution of background values (see Section 2.1.4).
- The distribution for background data may vary from one site to another, one environmental medium (e.g., soil, groundwater) to another, and one constituent to another. Background data may occur in the form of normal, lognormal, or other distributions, although it is expected that many background distributions will be (approximately) lognormal. The form of the data distribution should be considered in evaluating background values for each constituent, in each medium, at each site.
- BDL results are common for many constituents in background samples, and the frequency of BDL results may be much higher than for most compliance monitoring data sets. Therefore, specific methods for dealing statistically with BDL values (i.e., the regulation's default approach, assigning one-half of the detection limit to BDL values, or an alternative approach) should be identified (see Section 2.2).

4.3.2 Uses of Background in the Cleanup Standards Regulation

The distinction between natural background and area background values is important with respect to the uses of background data in the cleanup standards regulation (see the discussion in Section 4.2.2 above). Background data can generally be used in three ways to establish cleanup standards:

1. Natural background can be used to establish a cleanup standard for a hazardous substance for which no applicable or relevant and appropriate requirement (ARAR) or cleanup standard regulation value exists [WAC 173-340-704(2)(c)].

2. Natural background can be used to replace an existing Method A, Method B, or Method C cleanup standard when that standard is below the natural background level [WAC 173-340-700(4)(d)].

3. When Method A or Method B cleanup standards are below area background levels, Method C can be used to establish the cleanup standard. That cleanup standard may be equal to the area background value if it is within the allowable range for Method C standards, but the standard may not be greater than the maximum concentration allowable under Method C calculations [WAC 173-340-706(1)(a)].

Situations in which either natural or area background values will result in cleanup standards higher than those derived in Methods A, B, or C, based on ARARs or risk-equivalent calculations, may be infrequent. For many cleanup standard decisions, background values will not affect the cleanup standards. However, in cases where background values lead to the adoption of higher cleanup standards, this decision can be of great importance for reaching decisions on site cleanup. A flowchart presenting the role of background values in determining cleanup levels is shown in Figure 11.

4.3.3 Calculation of Background Values

4.3.3.1 General Issues—The uses of background data specified in MTCA regulations require that the distribution of background values (i.e., the varying concentrations reported within a set of background samples) be represented by a single selected value. That value will determine, for example, whether or not a numerical cleanup standard established under Method A, B, or C is below background.

The regulation states the following requirements [WAC 173-340-708(c),(d), and (e)]:

1. The statistical method used to evaluate available data shall be appropriate for the statistical distribution (e.g., lognormal) of each hazardous substance.

2. The lower tolerance limit may be used to compare a cleanup standard with background. That lower tolerance limit shall be based on a coverage of 95 percent and a tolerance coefficient of 95 percent (i.e., the background value shall be the lower 95 percent confidence limit on the 95th percentile of the background distribution).

3. Other statistical methods may be used if approved by Ecology.

4. Values below the method detection limit shall be assigned a value equal to one-half of the method detection limit. Values above the method detection limit but below the practical quantitation limit shall be assigned a value equal to the method detection limit. Alternative procedures for addressing not-detected values may be used if approved by Ecology.

Ecology has determined that the statistical procedures included in the regulation, including the use of lower tolerance limits, do not provide an appropriate method for evaluating background data and comparing cleanup standards to background. Therefore, alternative procedures are described in this guidance document. They are discussed in Section 4.3.3.2 below.

The same statistical methods are used for both natural and area background data, regardless of the intended uses of the data. The main features distinguishing natural and area background data sets under MTCA are the location and number of samples. The same locations are not equally representative of natural and area background conditions; therefore, any sampling plan for collecting background data should be carefully designed and reviewed with respect to the

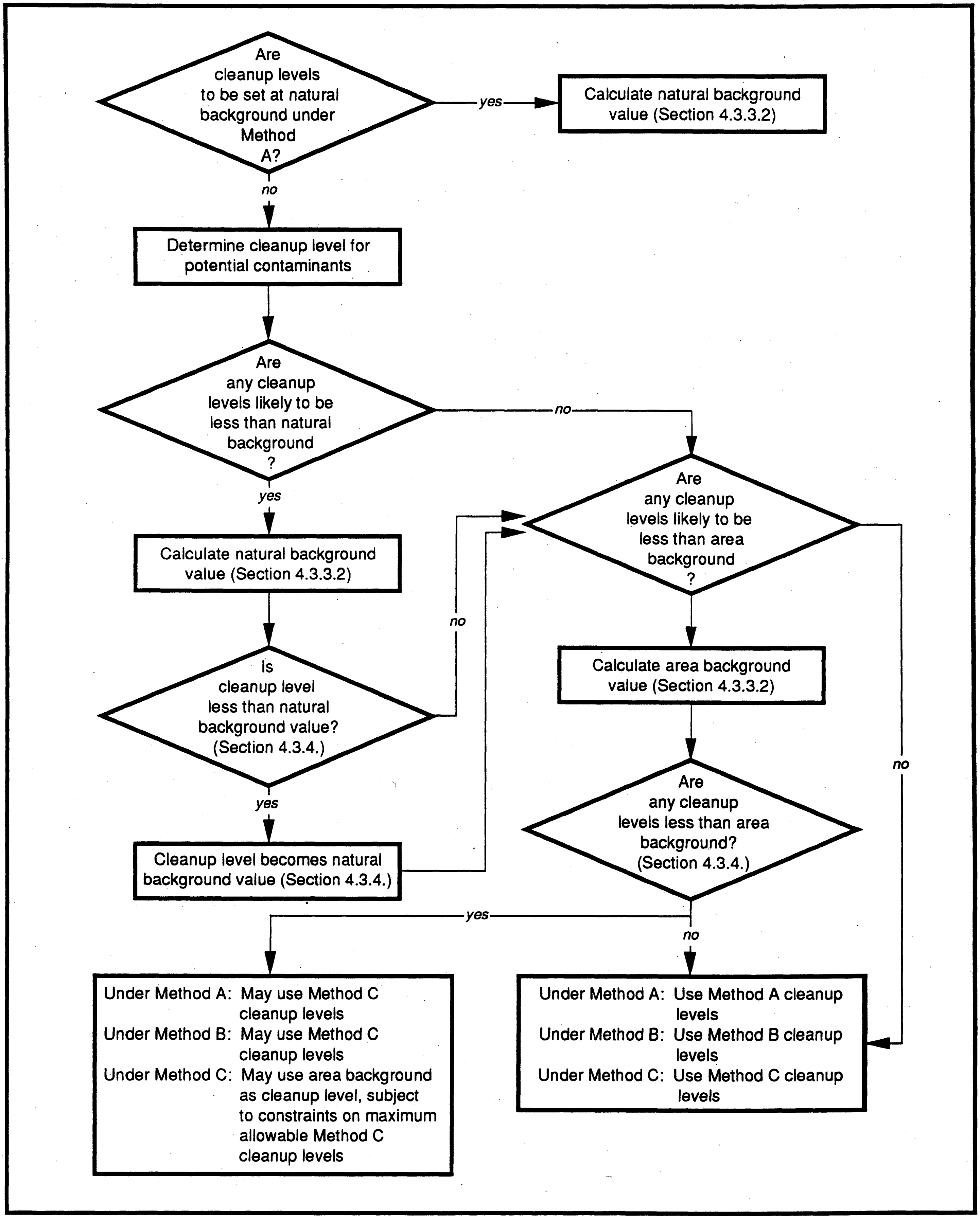


Figure 11. Flowchart demonstrating the role of background values in determining cleanup levels.

C704-19-09 0392

representativeness of those locations for the type of background data being sought. Area and natural background samples cannot be combined meaningfully in a single data set.

For soils data, the regulation specifies minimum numbers of background samples [WAC 173-340-708(11)(d)]. At least 10 soil samples are required to determine natural background levels, and at least 20 are required to determine area background levels. The minimum number of samples required for other media is not defined in the regulation and needs to be determined on a case-by-case basis. The minimum sample sizes of 10 or 20 samples may not result in data sets that provide accurate and representative estimates of background values (i.e., sampling errors may be relatively large). Estimates of upper percentile values of the background distribution may be particularly affected by small sample sizes. In many cases, it may be appropriate to collect a larger number of background samples to reduce possible sampling error effects and reach a better decision on cleanup standards.

The flowchart in Figure 12 provides an overview of the data evaluation procedures for determining possible cleanup standards based on background. Default procedures are shown in the left-hand column of Figure 12. The right-hand column provides for alternative methods. A numerical cleanup standard is still established, but it may be based on different data evaluation procedures. This could be as the result of site-specific characteristics, such as the form of the background data distribution, its coefficient of variation (CV) or degree of skew, the number of samples available, or other such factors.

The use of alternative procedures rather than the default procedures of Figure 12 for evaluations at MTCA sites will require submittal of adequate supporting information on the performance of the proposed tests (e.g., Type I and Type II error rates). Alternative procedures cannot be used unless they are reviewed and approved by Ecology.

4.3.3.2 Calculation Methods (Examples 9 and 10)—The default procedures for determining a cleanup standard based on background data are illustrated in Figure 12 and are discussed in this section. An abbreviated summary of the procedures shown in Figure 12 is provided in Supplement S-4. Statistical methods referred to in these default procedures are described in Sections 2, 5.2, and 5.3.

The default procedures result in a numerical value, calculated from site background data, that is used to represent background for evaluations of cleanup standards and compliance with background-based cleanup standards. Background data are assumed to be lognormally distributed; contrary assumptions shall not be made unless a lognormal distribution is statistically rejected at the 0.05 level. Lognormal distributions have a positive skew; this is often representative of data from environmental measurements, which are constrained on the low side by zero or the limits of analytical detection. Ecology performed computer analyses (called "Monte Carlo" simulations) to examine the performance (Type II error rates and power to detect residual contamination) of various percentiles of lognormal distributions as candidates for defining background cleanup standards. Those simulations included lognormal distributions with varying coefficients of variation (i.e., varying degrees of skew). Similar simulation evaluations were also performed for normal distributions. Based on the results of the Monte Carlo simulations, Ecology has selected the 90th percentile value as the default background value for cleanup

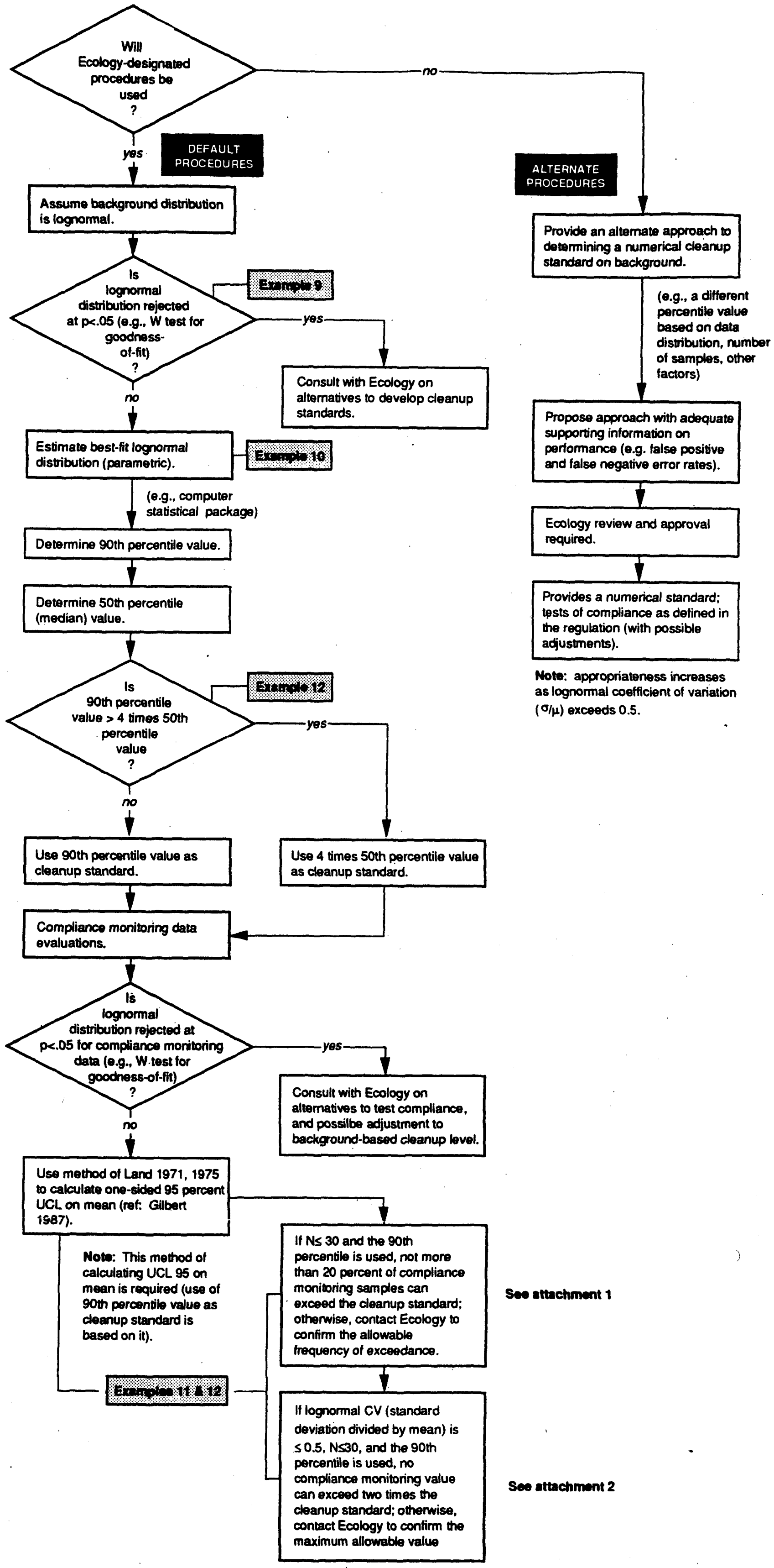


Figure 12. Flowchart for determination of cleanup standards based on background data.

C704-19-09 0392

TECHNICAL ATTACHMENT 1
TO FIGURE 12

ALLOWABLE FREQUENCY OF EXCEEDANCE OF CLEANUP STANDARDS BASED ON BACKGROUND
[Requires consultation with Ecology]

The cleanup regulations under MTCA include provisions for limiting the frequency of exceedances of a cleanup standard to no more than 10 percent. Where a cleanup standard is established based on risk estimates, ARARs, or other approaches at a level above background, the possibility of a "false positive" result does not arise. However, for a cleanup standard based on background, the possibility that exceedances of the standard occur as a result of chance alone (false positives) should be considered explicitly. This results in an adjustment to the allowable frequency of exceedances for background-based cleanup standards only.

A cleanup standard selected at a given percentile of the background distribution defines the probabilities of any single random sample from that background distribution being above or below the cleanup level. For example, a cleanup standard established as the 90th percentile (using the default procedures) of a background distribution results in a probability of 0.10 for a single sample exceeding the cleanup level, and 0.90 for that sample being less than the cleanup level. This "binomial" outcome leads directly to use of the binomial theorem to calculate probabilities of any frequency of exceedance of the cleanup level. Probabilities of exceedance depend only on the percentile chosen for background, the number of compliance monitoring samples, and the exceedance frequency.

Based on the percentile of background that the cleanup standard represents, let p and q represent the probabilities of a single random sample being greater than and less than (or equal to) the cleanup standard, respectively. For the default procedures where the cleanup standard is at the 90th percentile, as discussed above, p = 0.10 and q = 0.90. Let n be the number of compliance monitoring samples. Then, by the binomial theorem, the probability of exactly k out of the n compliance monitoring samples exceeding the cleanup standard is:

$$\text{probability} = p^k q^{(n-k)} \binom{n}{k}$$

where

$$\binom{n}{k} = [n(n-1)\ (n-2)\ldots(n-k+1)/k!$$

The last term on the right in the probability equation gives the number of different ways of selecting the k out of n samples that exceed the cleanup level (order of sampling not considered). Each such result has the same probability, namely $p^k q^{(n-k)}$. Using this equation to calculate the results for individual k values, the probability of k or more exceedances can easily be determined.

It should be recognized that there is a non-zero probability that none of the n samples exceeds the cleanup standard. For example, the probability that 0 of 10 compliance monitoring samples exceeds a cleanup standard based on the 90th percentile of background is just $(0.9)^{10} = 0.349$. Since some outcome must be observed, the sum of the probabilities from k = 0 to k = n must equal 1. Table A-5 provides binomial distribution results for selected values of p and n.

A 0.05-level false positive error probability is considered in the following analysis. With a discrete, rather than continuous, set of outcomes (only integral values are possible for k, the number of compliance monitoring results above the cleanup standard), an exact 0.05-level criterion is not available. The table below provides some illustrative results assuming that the cleanup standard is established at the 90th percentile of background:

Probability of k or more exceedances

k	number of samples: 10	15	20	25	30	40	50
3	.070	.184					
4	.013	.056	.133				
5		.013	.043	.098			
6			.011	.033	.073		
7				.010	.026	.100	
8						.042	
9						.016	.058
10							.025

An appropriate criterion for the allowable exceedances of a cleanup standard based on the 90th percentile of background can be determined from information on the probability of k or more exceedances out of n compliance monitoring samples. For example, with 20 compliance monitoring samples, the probability of 5 or more exceedances is 0.043 (approximately 0.05), and the maximum allowable number of exceedances is 4, or 20 percent.

Table A-5 may provide the information necessary for calculation of the probabilities of k or more exceedances in specific cases. To illustrate how to perform the necessary calculations, consider a case where the cleanup standard is established at the 80.22nd percentile of background, and 10 compliance monitoring samples are collected. Using the basic probability equation given above for the probability of exactly k out of n exceedances, a table of values can be simply computed. The values for p, q, and n are (1 - 0.8022), 0.8022, and 10, respectively. The initial values in such a table are:

Probability of exactly k out of 10 exceedances

k	probability	
0	0.1104	p^0q^{10}
1	0.2721	$p^1q^9(10)$
2	0.3019	$[p^2q^8(10)(9)]/2$
3	0.1985	$[p^3q^7(10)(9)(8)]/(3)(2)$
4	0.0857	$[p^4q^6(10)(9)(8)(7)]/(4)(3)(2)$

The probability of 5 or more out of 10 compliance monitoring samples exceeding a cleanup standard based on the 80.22nd percentile of background is 1 minus the sum of the tabled probabilities for k = 0, 1, 2, 3, or 4, or a probability of (1 - 0.9686) = 0.031. Thus, a maximum allowable number of exceedances would be 4 based on a 0.031-level false positive error rate.

NOTE: These tables are for illustrative purposes only. Contact Ecology for site-specific allowable exceedance.

TECHNICAL ATTACHMENT 2
TO FIGURE 12

ALLOWABLE MAGNITUDE OF EXCEEDANCE OF CLEANUP STANDARD BASED ON BACKGROUND
[Requires consultation with Ecology]

The cleanup regulations under MTCA include provisions for limiting the maximum magnitude of exceedance of a cleanup standard in a compliance monitoring data set to no more than two times the cleanup level. Where a cleanup standard is based on risk estimates, ARARs, or other approaches at a level above background, the possibility of a "false positive" result does not arise. However, for a cleanup standard based on background, the possibility that the maximum compliance monitoring value exceeds twice the cleanup level by chance alone (false positive) should be considered explicitly. This may result in an adjustment for the maximum allowable exceedance factor <u>for background-based cleanup standards only</u>.

The maximum allowable exceedance factor can be calculated to achieve a desired false positive error rate, for example 0.05, assuming that the background distribution is known. Under the standard default procedures, the background distribution is lognormal; the calculations illustrated here are for that distribution. The adjustment in the maximum factor of exceedance of a cleanup standard depends on the number of compliance monitoring samples, the shape of the lognormal distribution (determined by its coefficient of variation [CV], the standard deviation divided by the mean value for the distribution), and the percentile of background at which the cleanup standard is established.

For a given compliance monitoring sample size, n, a percentile of the distribution at which the probability of 1 or more exceedances is equal to 0.05 is calculated first. That probability is equal to 1 minus the probability of no exceedances. Let the percentile be denoted as (100 x q), so that q represents the probability of a single random sample being less than (or equal to) the percentile (see Attachment 1). Then

$$1 - q^n = 0.05$$

$$0.95 = q^n$$

and

$$(\log_e 0.95)/n = \log_e q$$

$$q = e^{(\log_e 0.95/n)}$$

The percentile of the distribution is then equal to 100q. For example, the value of 100q when there are 10 compliance monitoring samples is

$$100q = 100 \times e^{(\log_e 0.95/10)}$$

$$= 100 \times e^{(-0.0051)}$$

$$= 99.49$$

so there is a 5 percent chance of 1 or more out of 10 random samples from background exceeding the 99.49th percentile of the background distribution. The percentiles for 15, 20, and 30 compliance monitoring samples are the 99.66th, 99.74th, and 99.83rd, respectively.

Using information on the background distribution (i.e., the best-fit lognormal distribution under standard default procedures), the value at the percentile corresponding to this 0.05 false positive error rate can be estimated. This can be done using a computer statistical package such as STATGRAPHICS®, or by calculating percentiles using $\log_e$-transformed values and back-transforming to original units (e.g., see Example 10). The resulting value defines a criterion for limiting the maximum exceedance of the cleanup standard at a 0.05 false positive error rate.

A table of exceedance factors can be developed by calculating percentile values as described above and comparing them to cleanup standard values. Assuming a lognormal background distribution, default cleanup standard values will be at the 90th percentile up to a CV of about 1.5, and at 4 times the 50th percentile for CV values at or above 1.5. The results are as follows:

Maximum exceedance factor of cleanup standard (selected false positive rate = 0.05)

	number of samples:		
CV	10	20	30
0.1	1.14	1.16	1.18
0.2	1.29	1.35	1.39
0.3	1.46	1.56	1.62
0.4	1.65	1.79	1.89
0.5	1.84	2.04	2.18
0.6	2.05	2.31	2.49
0.7	2.26	2.60	2.83
0.8	2.49	2.90	3.19
0.9	2.71	3.21	3.56
1.0	2.94	3.52	3.94
1.5	4.10	5.19	6.01
2.0	6.56	8.66	10.27

These results illustrate that the maximum exceedance factor at a 0.05-level false positive error rate increases as either the number of compliance monitoring samples or the background distribution CV increases. At CV values above 1.5, the cleanup standard based on 4 times the 50th percentile value will also be lower than the 90th percentile; that difference in cleanup levels also increases the maximum exceedance factor.

Similar procedures can be used to determine a maximum exceedance factor for compliance monitoring samples in cases of other sample sizes, other CV values for a lognormal background distribution, cleanup standards at other than the default percentiles, or other types of known background distributions. For example, with 15 compliance monitoring samples, a lognormal background distribution with a CV = 0.7, and a cleanup standard at the 90th percentile, an exceedance factor of 2.46 results. For 10 compliance monitoring samples, a lognormal background distribution with a CV of about 3.65, and a cleanup standard at 4 times the 50th percentile (approximate 80.22nd percentile), an exceedance factor of about 16.5 results.

NOTE: This table is for illustrative purposes only; contact Ecology for site-specific allowable exceedance factor.

standard and site evaluations, subject to certain constraints discussed below. Section 2.1.2 discusses the estimation of percentile values.

The performance of the 90th percentile, especially with respect to Type II error rates (finding a site to be contaminated when it is really at background; Section 2.1.7), declines as the coefficient of variation (CV) of a background lognormal distribution increases. It is not known what CV values will characterize actual background data sets; many of them are expected to be only modestly skewed. It is noted that alternative procedures may become increasingly appropriate as the CV increases above about 0.5.

To address the possible significant increases in exposures and human health risks at 90th percentile background values, especially for strongly positively skewed background distributions, an additional evaluation measure is applied. Typical background values may be defined as at or near the 50th percentile value. The ratio of the 90th to the 50th percentile values for background is a measure of how far the potential cleanup standard value at the 90th percentile is above typical background levels. This ratio will be larger when the positive skew in the distribution is larger. As a matter of policy, Ecology constrains possible background cleanup standards to no greater than 4 times the 50th percentile concentrations. (This assumes that a risk-based cleanup standard based on Method A, B, or C is below the 50th percentile of background; if it is in fact above the 50th percentile, the limiting value for a background-based standard would be 4 times the Method A, B, or C cleanup level). Therefore, after a 90th percentile background concentration is determined, it is compared to a 50th percentile value and this ratio test is applied. In cases where 4 times the 50th percentile value is less than the 90th percentile value, this results in a lower background cleanup standard and a somewhat higher clean-site failure rate, balanced by lower potential exposures and human health risks.

The choice of the 90th percentile concentration of background for evaluation of cleanup standards and compliance actually depends on <u>both</u> the background and compliance monitoring data sets. If the background data are tested and rejected as lognormal (e.g., using the W test; see Section 2.1.4.1), the 90th percentile should <u>not</u> be used. A different percentile will be appropriate depending on the distribution of the background data. For example, Ecology simulations of background data sets drawn from a normal distribution indicate that the 80th percentile would be suitable in that case. The same percentile value does not result in the same performance (error rates and power to detect residual contamination) for different data distributions. If the background data are not lognormally distributed, Ecology should be consulted for alternative procedures. Example 9 illustrates a case where the background data appear to be normally rather than lognormally distributed. Example 10 addresses lognormal background data.

The choice of the 90th percentile when the background data <u>are</u> lognormally distributed is also contingent on the use of the method of Land (1971, 1975) for estimating an upper confidence limit on the mean of the compliance monitoring data distribution. That method is described in Section 5.2.1.2 and is only appropriate when the data are lognormally distributed. Therefore, if the compliance monitoring data are tested and rejected as lognormal, the 90th percentile may no longer be appropriate to use for background evaluations. Ecology should be consulted if background data are lognormally distributed but compliance monitoring data are not lognormal before proceeding with site compliance evaluations. The percentile defining background under MTCA could change in such cases from the default 90th percentile value.

In some cases, it may be appropriate to consider collecting additional background data to determine if the background distribution is really as skewed as suggested by an initial, small background data set. Any such additional background sampling should be carefully reviewed with Ecology before assuming that the data will be used in background evaluations. Higher background values should always be carefully reviewed to establish whether they could be influenced by a localized contaminant source (i.e., whether they are really representative of background).

Worksheet W-3 provides detailed instructions for calculating a background value for lognormally distributed data. Examples 9 and 10 provide comparisons of parametric and nonparametric methods for estimating percentiles of a distribution. When a specific distribution (e.g., lognormal or normal) is assumed for background, appropriate parametric methods should be used.

4.3.4 Establishing a Cleanup Standard from Background Data

After calculating an appropriate background value from a background data set, using the methods described in Section 4.3.3.2 above, the comparison of that value with a Method A, B, or C cleanup standard is straightforward. *It simply involves the comparison of two numbers.*

In the case of natural background comparisons, the higher of the two values will become the cleanup standard.

In the case of area background comparisons, a Method A or Method B cleanup standard that is greater than area background will still be used as the cleanup standard. If, on the other hand, area background is greater than the standard, then a Method C cleanup standard can be derived and used. The resulting standard may or may not be as large as the area background value.

For any comparison based on a given background data set, the results of the comparison may be accepted or additional background data may be collected, the background value recalculated based on a larger (pooled) data set, and the comparison re-evaluated. Collection of additional background data (sampling design, access agreements, sample collection, laboratory analysis, QA/QC review, and data validation) would normally require additional time. This should not be allowed to unnecessarily delay making site decisions. Schedule allowances for the possibility of a second round of background data collection should be considered early in the project.

The pooling of data collected in different time frames, and possibly involving different sampling procedures or analytical laboratories, also should be considered carefully prior to a second round of sampling. Sampling locations and sampling plans to be used for background characterization should be approved by Ecology. While the enhancement of a site background data set offers an opportunity for better characterization of background, and thereby better decision making, there are also statistical issues involving the post-hoc selection of a most-favorable data set for evaluation (continuing sampling until a favorable result is obtained and then stopping, introducing bias into the characterization process). Therefore, all background sampling should be carefully reviewed with Ecology.

4.3.5 Evaluating Compliance Monitoring Data When a Cleanup Standard is Based on Background (Example 11)

Once a numerical cleanup standard has been selected, whether based on risk-equivalent concentrations, ARARs, ecologically protective levels, natural or area backgrounds, or other criteria, the evaluation of compliance monitoring data with respect to the cleanup standard proceeds in exactly the same way. The fact that a numerical cleanup standard has been derived based on background data does not affect the types of evaluations of compliance monitoring data. However, some adjustments are required in the criteria based on the allowable frequency and magnitude of exceedance of a cleanup standard (see Section 5) when that standard is based on background. Those adjustments are discussed in this section, and are applicable <u>only</u> in the case of background-based standards.

The computer analysis performed by Ecology indicates that, for both the frequency and magnitude-of-exceedance criteria, evaluation of Type II error rates indicates that the criteria defined in the MTCA regulation should be adjusted when the cleanup standard is based on background. The probability of having more than 10 percent of the compliance monitoring samples above the 90th percentile of background is relatively high if the compliance monitoring data are from the background distribution (i.e., if the site is clean). Therefore, the criterion based on frequency of exceedance of the cleanup standard should be adjusted to a somewhat higher percentage. Attachment 1 to Figure 12 describes an adjustment procedure that should be used. For example, for relatively small compliance monitoring sample sizes ($n < 30$), not more than 20 percent of the samples should exceed a standard based on the 90th percentile background value. Consult Ecology for other cases (see Attachment 1).

An adjusted maximum allowable exceedance factor of the cleanup standard will depend on the number of compliance monitoring samples, the percentile used for a cleanup standard, and the CV of the lognormal distribution. Attachment 2 to Figure 12 describes how to determine a 0.05-level exceedance factor. For relatively small sample sizes and CV values, the usual criterion of no sample values more than two times the cleanup standard is still suitable. In other cases, a higher factor of exceedance is required. Attachment 2 provides details of the procedures for determining an appropriate factor for evaluating background-based standards (requires consultation with Ecology).

Evaluation of compliance monitoring data is the subject of Section 5.0. An illustration of such an evaluation based on a background cleanup standard is provided in Example 11. Appropriate adjustments to the frequency and magnitude-of-exceedance criteria are illustrated in Example 11. (See also Example 12).

4.4 GROUNDWATER CLEANUP STANDARDS BASED ON BACKGROUND DATA (Example 12)

Except for the requirement to assess compliance at each well or monitoring point [WAC 173-340-720(8)(c)(iv)], cleanup standards for groundwater are evaluated in almost exactly the same manner as those for soils or any other medium under MTCA. Therefore, the discussion for soils in section 4.3 above is equally applicable to groundwater. The only other issue for which the regulation treats soils and groundwater differently is the minimum number of background samples, which is specified for soils but left to a case-by-case determination for groundwater. Costs for collecting groundwater samples are typically much higher than for soil samples, often resulting in fewer groundwater samples being collected and smaller data sets being available for evaluation. The importance of the background data for site decisions should always be considered in addition to cost; the need for an adequate database may justify collecting more groundwater data, even at substantially increased costs.

The spatial and temporal aspects of groundwater variability are somewhat different than for soils and should be carefully considered in designing any background data collection program (see Section 5.3.5). Groundwater samples collected within reasonably small areas (i.e., close to the site) may not reflect the same groundwater population. Hydrogeologic and statistical information should be considered in evaluating the representativeness of groundwater samples for defining a background value related to site conditions. It is not necessary that samples be from hydraulically connected locations, but it is necessary that they be from representative locations.

Background groundwater concentrations, as well as onsite concentrations, may also vary substantially over time (e.g., seasonally). This may be particularly important for comparing compliance monitoring data and background-based cleanup standards. Seasonal variation—for example as influenced by different precipitation and infiltration rates throughout the year–should not be confounded with differences between site and background concentrations.

Both spatial and temporal components of variation in groundwater concentrations should be carefully evaluated as part of the design of any sampling program, but especially for background sampling where the data will be used for cleanup standards decisions. In general, multiple samplings from the same well cannot be used to increase sample size unless a demonstration can be made that repeat measurements at individual wells are not significantly correlated temporally. Any such demonstration should address temporal and spatial variability independently.

An example of the development of a groundwater cleanup standard based on background data is provided as Example 12. Additional considerations for groundwater are discussed in Section 5.3.5.

4.5 SURFACE WATER CLEANUP STANDARDS [RESERVED]

4.6 AIR QUALITY STANDARDS [RESERVED]

4.7 SEDIMENT STANDARDS [RESERVED]

5. ASSESSMENT OF COMPLIANCE MONITORING DATA FOR MEETING CLEANUP STANDARDS

5.1 DECISION-MAKING PROCESS

After the cleanup standard has been determined (see Section 4), the data from the site must be evaluated to determine whether the exposure unit meets the cleanup standard. This decision is independent of the approach used to define the numerical cleanup standard. As described in Section 4, the cleanup standard may be based on applicable state and federal laws, risk, ecological factors, or analytical considerations (e.g., BDL data, PQL), or may be related to background levels of the contaminant near the site. In all cases, a single numerical value is obtained for the cleanup standard, to which site data can be compared. The process involved in making the decision as to whether the exposure unit meets cleanup standards for soils and groundwater is shown in Figures 13 and 14, respectively. Note that the criteria for allowable frequency and maximum magnitude of exceedance of cleanup standards may be adjusted in consultation with Ecology if the cleanup standard is based on background, as described in Section 4, and covered in Figure 12.

If issues at a particular site become more complex than those covered in this document, additional assistance should be sought from a statistician or Ecology.

5.2 COMPARING SITE DATA TO SOIL CLEANUP STANDARDS

Two methods for demonstrating that the site meets the cleanup standards are recognized: a method using a confidence interval, and a parametric method for percentiles. The MTCA regulations state:

> For cleanup levels based on short-term or acute toxic effects on human health or the environment, an upper percentile soil concentration shall be used to evaluate compliance with cleanup levels [WAC 173-340-740(5)(c)(iv)(A)].

and

> For cleanup levels based on chronic or carcinogenic threats, the mean soil concentration shall be used to evaluate compliance with cleanup levels... [WAC 173-340-740(5)(c)(iv)(B)]

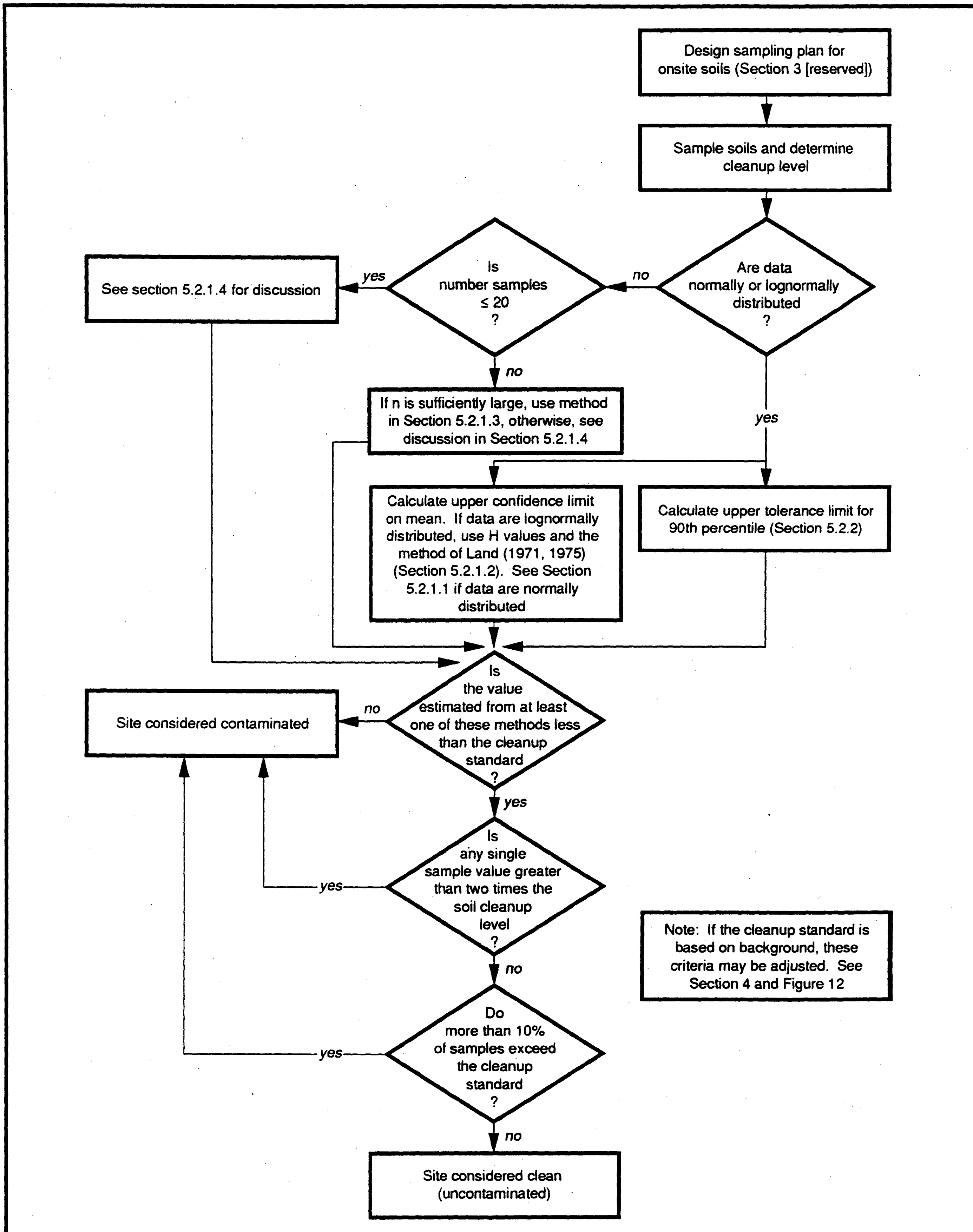


Figure 13. Flowchart for determining if soils at a site meet a cleanup standard.

C704-19-09 0392

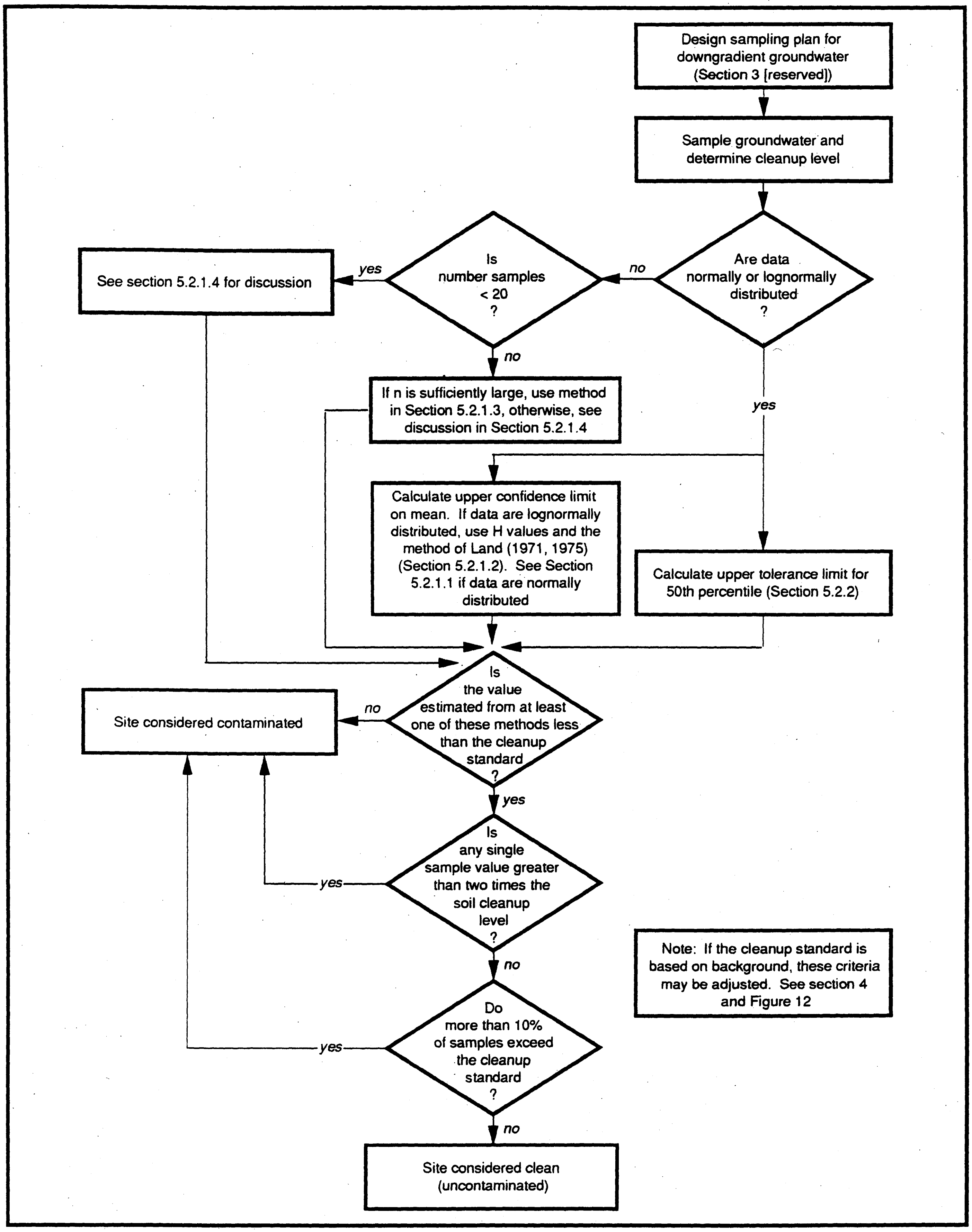


Figure 14. Flowchart for determining if groundwater at a site meets a cleanup standard.

Thus, the confidence interval approach (Section 5.2.1) should be used for cleanup levels based on chronic or carcinogenic effects, and the tolerance interval approach (Section 5.2.2) should be used for cleanup levels based on short-term or acute toxic effects. Also acceptable are "other statistical methods approved by the department" [WAC 173-340-740(7)(d)].

The confidence interval and tolerance interval methods should not be performed on data that cannot be approximated by a normal or lognormal distribution. A distribution-free (non-parametric) method should be used for this type of data. Nonparametric confidence interval estimates are described in Section 5.2.1.3 and 5.2.1.4 below.

5.2.1 Evaluation of Compliance Monitoring Data Based on Upper Confidence Limit on the Mean

The MTCA soil compliance monitoring regulations state that an appropriate statistical method for evaluating compliance (for cleanup levels based on chronic or carcinogenic effects) is "a procedure in which a confidence interval for each hazardous substance is established from site sampling data and the soil cleanup level is compared to the upper confidence level [WAC 173-340-740(7)(d)(i)], and "statistical tests should be performed at a Type I error level of 0.05" [WAC 173-340-740(7)(e)(i)]. Thus, for soils, compliance monitoring requires estimating the 95-percent confidence interval about the mean, and comparing this value to the cleanup level.

The method for determining whether an exposure unit meets the cleanup level is to compare the upper confidence limit (UCL) of the site data with the cleanup level. This method should be used for most cleanups; the tolerance interval method (Section 5.2.2) should be used when the cleanup level is based on short-term or acute toxic effects on human health or the environment. The procedure for calculating the UCL is discussed below.

5.2.1.1 Normally Distributed Data—The sample mean determined from a set of samples from a normal distribution provides a point estimate of the population mean. Different compliance monitoring data sets from the same site would usually result in somewhat different sample mean values. This indicates that the sample mean itself has a probability distribution. Confidence intervals for the mean are based on the distribution of the sample mean. The sample mean follows a Student's t distribution.

One-sided confidence interval values for the Student's t parameter are provided in Table A-4. The procedure for calculating a one-sided upper confidence limit for the mean for data from a normal distribution is as follows:

1. Calculate the mean ($\bar{x}$) and standard deviation (s) of the compliance monitoring data.

2. In Table A-4, look up the appropriate t value. For a one-sided 95-percent confidence interval ($\alpha = 0.05$), the column headed .05 is used. The t value is determined by finding the row corresponding to the degrees of freedom (df), which is one less than the number of samples, n.

$df = n - 1$

3. The upper confidence limit (UCL) for the mean is

$$UCL = \bar{x} + t_{1-\alpha,n-1}\frac{s}{\sqrt{n}}$$

where

$\bar{x}$ = sample mean

s = sample standard deviation

n = number of compliance monitoring samples

t = value of the t parameter from Table A-4, based on a one-sided α of 0.05 and n-1 degrees of freedom.

5.2.1.2 Lognormally Distributed Data—A method for calculating the upper one-sided confidence limit for the mean of a lognormal distribution is provided by Land (1971, 1975). This method is also described in Gilbert (1987). The procedure uses statistics calculated from the $\log_e$-transformed sample data from a lognormal distribution, as well as a parameter, H, determined from tabled values.

For a 95-percent one-sided confidence interval (α = 0.05), the upper confidence limit is calculated by

$$UCL = \exp(\bar{y} + 0.5s_y^2 + \frac{s_y H_{1-\alpha}}{\sqrt{n-1}})$$

where

exp = e raised to the indicated power

$\bar{y}$ = mean of the $\log_e$-transformed data

s_y = standard deviation of the $\log_e$-transformed data

n = number of compliance monitoring samples

α = significance level (0.05)

H = tabled H value from Figure A-1 (in the Appendix).

The value of the parameter H depends on the number of compliance monitoring samples, n, and on the variability of the sample data, measured by the standard deviation of the $\log_e$-transformed data, s_y. Land (1971, 1975) provides tabled H values.

Figure A-1 in Appendix A and Supplement S-2 give nomographs of selected H values for calculating one-sided 95-percent upper confidence limits for the lognormal mean. The approximate values that can be read off that nomographs will often support a determination of whether the UCL on the lognormal mean is greater than or less than the cleanup standard. For more accurate H values, tabled values (Land 1971, 1975) should be consulted. Land (1975) indicates that cubic interpolation (four-point Lagrangian interpolation) should be used to interpolate additional H values from the tables; however, this is complex, and in practice, the simpler linear interpolation will often suffice.

Detailed instructions for calculating the one-sided 95-percent upper confidence limits for the lognormal mean using Land's method are provided in Worksheet W-2.

5.2.1.3 Other Distributions with Large Sample Size—[*Requires consultation with Ecology.*] If compliance monitoring data indicate that both the normal and lognormal distributions should be rejected (e.g., by the W test), it may be possible to find another known distribution that is not rejected by an appropriate goodness-of-fit test. There may be procedures in the statistical literature for estimating upper confidence limits for the mean of those other known distributions, or for defined transformations of the distributions. If such methods exist, they may allow calculation of a UCL for the mean of the compliance monitoring data. Generally, however, compliance monitoring data that are neither normal nor lognormal will not have explicit methods for calculating a UCL for the mean. In most cases, the distribution of the data may be unknown.

A method providing approximate one-sided upper confidence limits for the mean for "sufficiently large sample sizes," n, from any distribution is based on the normal distribution and is described in Gilbert (1987; see p.139). As Gilbert (1987, p. 140) states, "There appears to be no simple rule for determining how large n should be for [this equation] to be used. It depends on the amount of bias in the confidence limits that can be tolerated and also on the shape of the distribution from which the data have been drawn. If the distribution is highly skewed, an n of 50 or more may be required."

The approximate one-sided upper 95 percent confidence limit for the mean is calculated by

$$UCL = \bar{x} + Z_{1-\alpha}\frac{s}{\sqrt{n}}$$

where

$\bar{x}$ = sample mean

s = sample standard deviation

n = number of compliance monitoring samples

$Z_{1-\alpha}$ = value of the Z parameter from the normal distribution for a defined α level. For a one-sided upper 95 percent confidence limit, a value for Z_{95} determined from Table A-6 is 1.645.

5.2.1.4 Other Distributions with Small Sample Size —[*Requires consultation with Ecology.*] In some cases, it may be apparent even from a small data set that neither the lognormal nor normal distribution is appropriate. For example, the data may be strongly bimodal due to the inclusion of values from a hot spot. For most sites, the number of compliance monitoring samples per exposure unit or exposure unit for which a cleanup decision is required will be relatively small compared to the sample size that might support use of the approximate method described in Section 5.2.1.3 above for calculating a UCL for the mean. Reliable statistical methods do not exist for estimating a UCL for the mean from unknown distributions where only a small number of samples are available.

In some cases, a different statistical test (e.g., upper tolerance limit test for a percentile of the distribution) may also be appropriate for use under the MTCA regulations, and that test could be used in place of one based on the UCL for the mean. Procedural options if a UCL for the mean is needed include the following:

1. Use the approximate procedure described in Section 5.2.1.3 even though the sample size is small. The likelihood that a substantial bias is introduced in the UCL estimate because of sampling error will increase as the number of samples decreases.

2. Develop a larger compliance monitoring data set for evaluation. The larger data set would have to be collected and evaluated based on a sampling plan reviewed and approved by Ecology. The larger compliance monitoring data set could support an assumption of a normal or lognormal distribution where the smaller initial data set did not; failing that, it would still result in a better approximation using the methods of Section 5.2.1.3. The cost of additional compliance monitoring data collection should be considered in comparison to the potential consequences of a poor site cleanup decision based on a small sample size.

3. Evaluate an upper tolerance limit for a percentile selected on a site-specific basis by Ecology instead of an upper confidence limit for the mean. The percentile would be

selected to reflect the approximate estimated location of the mean based on the sample results.

Small sample sizes with single, uncomposited samples will unavoidably result in difficulties for statistical evaluations of the likely true mean of a constituent at a site. For alternatives using an approach with a relatively small number of composited samples, see Neptune et al. (1990) and Ryti and Neptune (1991).

5.2.2 Evaluation of Compliance Monitoring Data Based on Upper Tolerance Limit for the 90th Percentile

For cleanup levels based on short-term or acute threats, an appropriate statistical method is "a parametric test for percentiles based on tolerance intervals to test the proportion of soil samples having concentrations less than the soil cleanup level" [WAC 173-340-740-(7)(d)(ii)]. In addition, "the true proportion of samples that do not exceed the soil cleanup level shall not be less than ninety percent. Statistical tests shall be performed with a type I error level of 0.05" [WAC 173-340-740-(7)(f)(ii)]. Thus, for soils, MTCA requires a 95-percent confidence interval (Type I error level of 0.05) around the 90th percentile [WAC 173-340-740-(7)(f)(iii)].

5.2.2.1 Normally Distributed Data—Tolerance limits are defined in Section 2.1.10. An upper tolerance limit for a percentile is much like a one-sided confidence interval for that percentile, and tolerance limits are used within MTCA as a method of taking possible sampling error into account (the point estimates derived from the data may not accurately reflect the underlying population value for the percentile).

An upper tolerance limit is calculated using sample statistics for the mean ($\bar{x}$) and standard deviation (s) and tabled values for a parameter, k. That parameter depends on the percentile of interest; the number of samples, n; and the "coverage" of the tolerance interval (equivalently, the α level for the one-sided confidence interval). Values for k for calculating 95 percent upper tolerance limits ($\alpha = 0.05$) for selected percentiles of a normal distribution are given in Table A-3. In that table, percentiles are identified by the value of P_o = (1 - percentile/100). Thus, the 90th percentile is represented by a P_o value of 0.10.

An upper 95 percent tolerance limit (T_U) for the 90th percentile of the compliance monitoring distribution is determined by

$$T_U = \bar{x} + ks$$

where

$\bar{x}$ = sample mean

s = sample standard deviation

k = value determined from table A-3 for the tolerance limit parameter, k, with $\alpha = 0.05$, n equal to the number of compliance monitoring samples, and P_o equal to 0.10 for the 90th percentile

This method can be used only when the data are normally distributed.

5.2.2.2 Lognormally Distributed Data—Both percentiles and upper tolerance limits for percentiles from a lognormal distribution can be estimated by first transforming the data (using $\log_e$), calculating values on the normally distributed transformed data, and then back-transforming to original units (raising e to the power of the result calculated from the transformed data).

An upper tolerance limit for the 90th percentile of a lognormally distributed compliance monitoring distribution is calculated as follows:

1. Transform the raw compliance monitoring data using a $\log_e$ transformation.

2. Using the method described in Section 5.2.2.1 above for normally distributed data, calculate an upper tolerance limit for the transformed data. Let the result be designated as T_y.

3. The upper tolerance limit for the 90th percentile of the lognormal distribution is then

$$T_U = \exp(T_y)$$

where

exp = e raised to the indicated power

T_y = upper tolerance limit calculated for the $\log_e$-transformed data.

5.2.2.3 Nonparametric Methods for Upper Confidence Limit with 20 or Fewer Samples—[*Requires consultation with Ecology.*] Regardless of the form of the distribution, nonparametric methods can be used to estimate an upper confidence limit for percentiles of the distribution. For sample sizes of 20 or fewer, a method described in Conover (1980) can be used. That method is discussed in this section. For sample sizes greater than 20, a method described in Gilbert (1987) can be used. It is described in the next section.

For sample sizes less than 20:

1. Use Table A-5 to find tabled values for b at approximately $\alpha/2$ and $1-\alpha/2$. The method for using Table A-5 is to read across the table for the percentile of interest (in this case, $p = 0.90$), and down the left-hand column for the value of n. Then move down the entries corresponding to different y values (which refer to the number of

occurrences of a binomial variable in N trials) until the entry in the table (b) is approximately equal to $\alpha/2$. find the corresponding value of y in the far left column. Add 1 to this value to get r.

2. Continue down the column until you reach an entry approximately equal to $1-\alpha/2$. Find the corresponding value of y in the far left column. Add 1 to this y value to get s.

3. Order the data from smallest to largest, and assign a rank (y value) to each value. If two or more data points have the same value, order them consecutively, and assign each its own rank. Determine the data value corresponding to s and r. These values represent the upper and lower confidence limits about the percentile of interest.

4. Compare the upper confidence interval with the cleanup standard. If the upper confidence limit is greater than the cleanup standard, the site is considered to be contaminated.

Example 15 provides a numerical demonstration.

5.2.2.4 Nonparametric Methods for Upper Confidence Limit with More Than 20 Samples—[*Requires consulation with Ecology*] For sample sizes greater than 20, a nonparametric method described by Gilbert (1987, p. 142) can be used to estimate one-sided upper confidence limits:

1. Find the value for $Z_{1-\alpha}$ in Table A-6, where $Z_{1-\alpha}$ = percentile of normal distribution.

2. Calculate

$$u = p(n+1) + Z_{1-\alpha}[np(1 - p)]^{1/2}$$

where

u = rank of upper confidence limit

p = percentile

n = number of samples.

3. Order the data from smallest to largest, and assign a rank to each value.

4. If u is an integer, then the data value corresponding to that rank is the upper confidence limit. If u is not an integer, the limit must be obtained by linear interpolation

between the two closest values. See Example 5 for a demonstration of linear interpolation.

5. Compare the upper confidence interval with the cleanup standard. If u is greater than the cleanup standard, the site is still considered to be contaminated.

5.2.3 Additional Requirements for Determining if a Site is Clean

In addition to comparing site data to the cleanup standard, there are two other requirements that must be met before a site can be determined to be "clean" [WAC 173-340-740(7)(e) and (f)]:

1. No single sample concentration shall be greater than two times the soil cleanup level.

2. Less than 10 percent of the sample concentrations shall exceed the soil cleanup level.

For background-based cleanup standards, the adjustments to these criteria (discussed in Section 4.3) should be considered.

5.3 COMPARING SITE DATA TO GROUNDWATER CLEANUP STANDARDS (EXAMPLE 17)

Statistical requirements for groundwater [WAC 173-340-720(8)] are similar to those for soil, except that the parametric method for percentiles requires a 95 percent confidence interval on the <u>50th percentile</u> (i.e., the median). Note also that compliance with a cleanup standard must be determined for each well or monitoring point [WAC 173-340-720(8)(c)(iv)], while compliance decisions for soil are normally based on combined data from different sampling points.

5.3.1 Normally Distributed Data

For a normal distribution, the median is equal to the mean. Therefore, the methods described in Section 5.2.1.1 for estimating a one-sided upper confidence limit of the mean can be used to evaluate compliance monitoring data based on the median.

5.3.2 Lognormally Distributed Data [*Requires consultation with Ecology.*]

A method for estimating the approximate two-sided confidence interval for the true median of a lognormal distribution is given by Gilbert (1987; see p. 173).

An upper confidence limit for the 100(1-α) percent two-sided confidence interval for the median of a lognormal distribution is calculated using the $\log_e$-transformed compliance monitoring data. First calculate the arithmetic average and standard deviation of these transformed data, y and s_y, respectively. The one-sided upper 95-percent confidence limit is then estimated, using $\alpha = 0.10$ for the two-sided equation given in Gilbert (1987), by:

$$UCL = \exp(\bar{y})\exp(t_{1-\alpha,n-1}\frac{s_y}{\sqrt{n}})$$

where

exp = e raised to the indicated power

y = mean of the $\log_e$-transformed data

s_y = standard deviation of the $\log_e$-transformed data

t = tabled value of the t distribution from Table A-4 (note that since this is a one-sided table, the column heading at α 0.05 level is used)

n = number of samples.

As discussed in Gilbert, this estimate is biased high, but the amount of bias decreases with increasing sample size and is generally small unless the skew of the lognormal distribution (i.e., its coefficient of variation) is very large.

5.3.3 Nonparametric Method for Upper Confidence Limit [*Requires consultation with Ecology.*]

A nonparametric method for providing confidence limits for the median of any continuous distribution is provided by Van der Parren (1970; see Appendix A). This method can be used regardless of the distribution of the compliance monitoring data. It provides confidence intervals that are equal to selected ranked data values; the confidence interval coverage is approximate rather than exact.

To determine an upper confidence limit for the two-sided confidence interval for the population median, the following procedure is used:

1. Sort the data from lowest to highest values and assign ranks, increasing with concentration.

2. From Table A-7 (extracted from the original Van der Parren reference for α equal to 0.05) find the value of j, the rank corresponding to the estimated upper confidence limit, for the sample size n.

3. Determine the concentration for the j^{th}-ranked compliance monitoring data value.

4. That concentration is an estimated upper confidence limit for an approximate α level of 0.05. The actual α level can be determined from tabled values given in the Van der Parren reference (attached in Appendix A).

Other tests are discussed Gilbert (1987, Chapters 11, 16, and 17) and may be appropriate for confirmatory analysis.

5.3.4 Additional Requirements for Determining if a Site is Clean

There are two other requirements that must be met before groundwater at a site can be determined to be "clean" [WAC 173-340-740(7)(e) and (f)]:

1. No single sample concentration shall be greater than two times the groundwater cleanup level.
2. Less than 10 percent of the sample concentrations shall exceed the groundwater cleanup level during the representative sampling period.

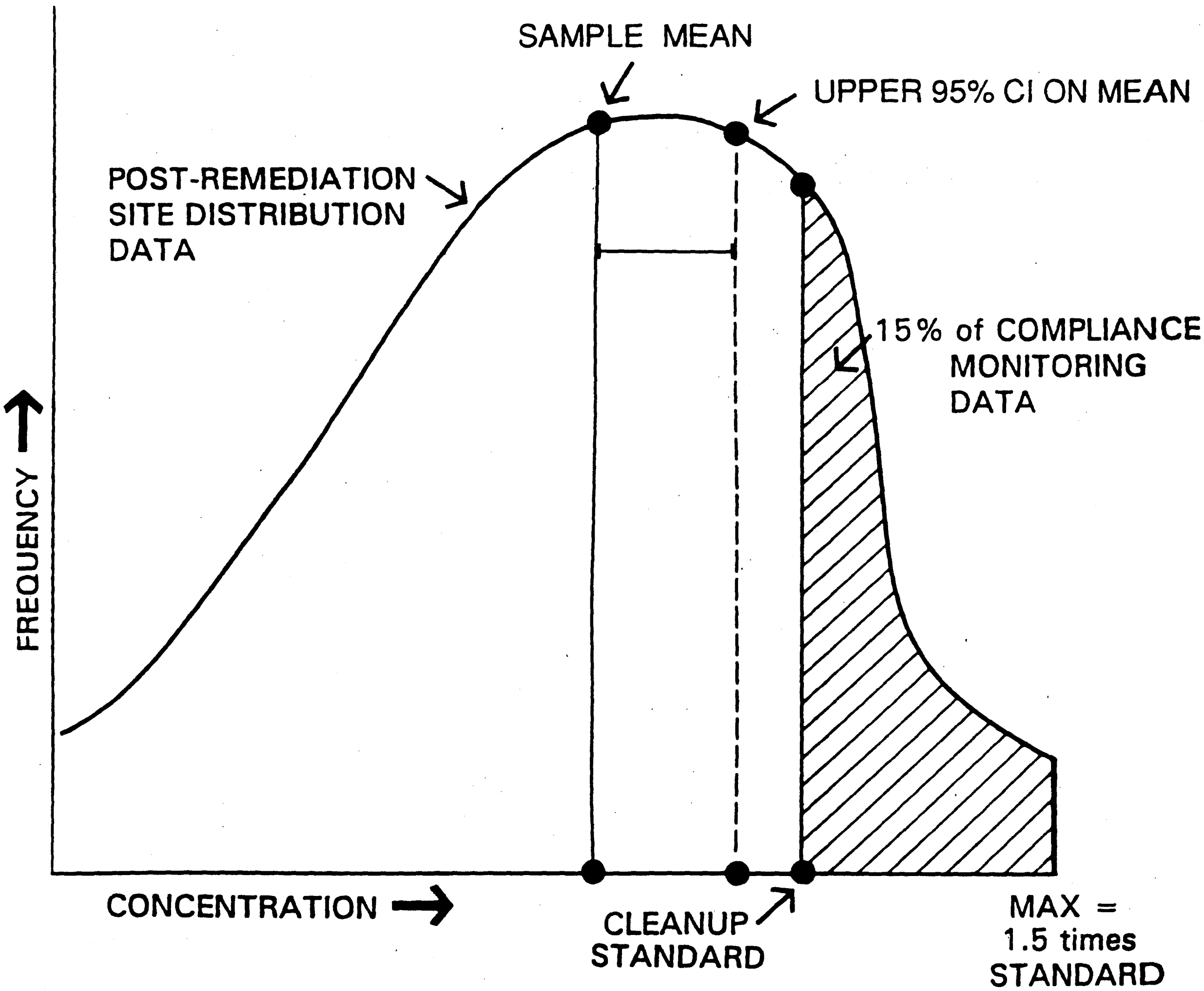
For background-based cleanup standards, the adjustments to these criteria (discussed in Section 4.3) should be considered.

Figure 15 shows the relation between these requirements using a hypothetical probability distribution from a site. This figure shows that the site data meet two of the three requirements for the site to be considered clean: the 95-percent CI on the median is below the cleanup standard, and no single sample concentration is greater than two times the cleanup level. However, 15 percent of the sample concentrations exceed the cleanup standard. Therefore, the site does not meet all three criteria and must be considered contaminated.

5.3.5 Additional Considerations for Groundwater

The following topics are not described in MTCA, but are discussed here because they may be significant issues at some sites. If it appears that these issues are relevant to a site, and the discussion here is not sufficient, additional assistance should be obtained from references listed, a statistician, or Ecology.

5.3.5.1 Trends—Groundwater is typically monitored for two purposes: 1) to determine contaminant concentrations of potentially impacted groundwaters relative to non-impacted (background) groundwater, and 2) to determine trends in concentrations with time or location, or both. Statistical methods must be applied to determine if temporal or spatial variability in contaminant concentrations is significant. If significant variations are detected, statistical methods can be applied to determine if the variations indicate verifiable trends.



In this example, the site meets only two of three criteria.

1) Is the upper CI on the mean less that the cleanup standard? Yes.
2) Is the maximum compliance monitoring value less than 2x standard? Yes.
3) Are 10 percent of the data above standard? No. 15 percent of the data are above standard. The site is not clean enough.

Figure 15. Conceptual basis for answering the question "Is the groundwater at the site clean enough?"

Contaminant concentrations measured at one location may vary naturally with time. This variation may be entirely random, it may follow a predictable trend or cycle, or it may have a random distribution superimposed upon a predictable trend or cycle. Variability in contaminant concentrations may be due to cyclic or non-cyclic changes in water-table elevations (tides, river stage changes, precipitation, and seasonal changes in infiltration rates and temperatures). Changes in concentrations may indicate effects of an onsite release, or they may reflect natural or anthropogenic regional changes.

To evaluate temporal trends (steady increases or decreases in contaminant concentrations), upgradient monitoring should be performed over a period of at least one year, because regression methods can yield misleading data when only a portion of an annual cycle is considered. Therefore, it is important for data to be collected over a period sufficient to establish cyclical trends. Occurrence of trends can be determined by plotting analyte concentrations vs. time and visually inspecting the plot to determine whether seasonal fluctuations are apparent. In addition, a linear regression can be fitted to contaminant concentrations vs. time, and a *t*-test performed to determine if the slope of the regression line is significantly different than zero (Gilbert 1987). Although it was previously stated that the *t*-test is not usually applicable at MTCA sites because it is inconsistent with the null hypothesis that the "site exceeds cleanup levels," it is applicable in this instance because the null hypothesis is that the slope of the line is <u>not different</u> from zero. This null hypothesis can be tested using conventional statistical methods. The *t*-test is described in many introductory statistical textbooks.

If seasonal trends are present in the data, it is critical that background contaminant concentrations measured during a particular period are compared to downgradient data from the same period. For example, suppose concentrations of a particular contaminant tend to decrease in the summer and increase in the winter. If the background concentrations are measured in the summer, and then compared to onsite concentrations measured in the winter, it may appear that the site is contaminated when the data really reflect only seasonal variation. Clearly this is not desirable, because remediation could be required on a site that is, in fact, "clean."

5.3.5.2 Serial Correlation—Most standard statistical tests assume that the data are independent. This means that there is no correlation between the data: that the chance of measuring a high or low concentration in a well is the same for each well at all times. However, if a well is sampled one day and then sampled again the next day, it is likely the concentration will be similar for each day. This is known as serial correlation—the linear dependence between observations in time. Even wells sampled on a quarterly basis can exhibit serial correlation (Montgomery et al. 1987). Such data violate the assumption of independence, without which the use of many statistical techniques may be precluded. A thorough discussion of serial correlation is beyond the scope of this document. However, Montgomery et al. (1987) suggest using statistical techniques that are insensitive to serial correlation or averaging the data over time periods sufficiently large that the serial correlation is insignificant. They also describe methods for determining whether data are serially correlated, which would indicate the length of a "sufficiently large" time period. If serial correlation appears to be a problem at a particular site, further statistical assistance should be sought.

5.3.5.3 Period of Time for Determining Background Concentrations—The period of time must be defined during which upgradient (background) data will be used for comparison with data collected onsite. The period considered may be prior to operation of the site or the start of onsite monitoring. It may also be a moving window (e.g., as one year prior to each monitoring event). In some cases, use of all available background data is the preferred method, because the environment is protected from short-run fluctuations that may dominate a moving window, while the potentially liable person (PLP) is likely to have increased confidence in the interpretation afforded by an increased size in the background data set (Gibbons 1990). However, use of all data may decrease the power to detect increases in groundwater contamination. The decision to use all or part of the data should be based on a consideration of the consequences of each detection.

5.4 COMPARING SITE DATA TO SURFACE WATER STANDARDS

MTCA states that when "surface water cleanup levels are based on requirements specified in applicable state and federal laws, the procedures for evaluating compliance that are specified in those requirements shall be utilized to evaluate compliance with surface water cleanup levels unless these procedures conflict with the intent of this section" [WAC 173-340-730(7)(d)]. "Where procedures for evaluating compliance are not specified in an applicable state and federal law, the statistical methods used to evaluate compliance with surface water cleanup levels shall be appropriate for the distribution of the hazardous substance sampling data" [WAC 173-340-730(7)(e)]. The confidence interval and tolerance interval procedures described above are appropriate tests. The tolerance interval procedure, however, requires a decision about the percentile to be used and can be used only with normally distributed data. If the data are not normally distributed, transformation (e.g., by converting to logarithms) may correct this. Alternatively, the groundwater guidance document cited above includes a nonparametric test for proportions (U.S. EPA 1988, p. 5-21) that does not require normally distributed data. Other tests described in that guidance document, such as regression analysis, may be useful in situations where surface water contaminant concentrations are changing over time.

5.5 COMPARING SITE DATA TO AIR QUALITY STANDARDS

Requirements are given in WAC 173-340-750(7). Consult staff in the Department of Ecology Air Program for technical assistance.

6. GEOSTATISTICS [RESERVED]

BIBLIOGRAPHY[1]

Abramowitz, M., and I.A. Stegun. 1964. Handbook of mathematical functions with formulas, graphs, and mathematical tables. Applied Mathematics Series. National Bureau of Standards. Vol. 55.

Cohen, A.C., Jr. 1959. Simplified estimators for the normal distribution when samples are singly censored or truncated. Technometrics. 1:217–237.

Cohen, A.C., Jr. 1961. Tables for maximum likelihood estimates: singly truncated and singly censored samples. Technometrics. 3:536–541.

Conover, W.J. 1980. Practical nonparametric statistics. Second Edition. John Wiley and Sons, New York, NY. 493 pp.

D'Agostino, R.B. 1971. An omnibus test of normality for moderate and large size samples. Biometrika 58:341–348.

Davis, C.B., and R.J. McNichols. 1987. One-sided intervals for at least p of m observations from a normal population on each of r future occasions. Technometrics. 29:359–370.

Duncan, A.J. 1962. Bulk sampling: Problems and lines of attack. Technometrics 4(2):319–344.

Elder, R.S., W.O. Thompson, and R.H. Myers. 1980. Properties of composite sampling procedures. Technometrics 22(2):179–186.

Finkel, A.M. 1990. A simple formula for calculating the "mass density" of a lognormally distributed characteristic: Applications to risk analysis. Risk Analysis. 10:291–301.

Gibbons, R.D., 1990. A general statistical procedure for groundwater detection monitoring at waste disposal facilities. Ground Water, 28(2):235–243.

Gilbert, R.O. 1982. Some statistical aspects of finding hot spots and buried radioactivity. In: TRAN-STAT: Statistics for environmental studies, No. 19. Pacific Northwest Laboratory, Richland, WA. PNL-SA-10274.

Gilbert, R.O. 1987. Statistical methods for environmental pollution monitoring. Van Nostrand Reinhold Company, New York, NY. 320 pp.

[1] Many of these references were cited in the text. Others are provided as sources for additional information.

Gilliom, R.J:, and D.R. Helsel. 1986. Estimation of distributional parameters for censored trace level water quality data: 1. Estimation techniques. Water Resources Research. 22:147-155.

Goodman, I. 1987. Graphical and statistical methods to assess the effect of landfills on groundwater quality. University of Wisconsin. Madison, WI.

Haas, C.N., and P.A. Scheff. 1990. Estimation of averages in truncated samples. Env. Sci. Technol. 24:912-919.

Helsel, D.R. 1990. Less than obvious: Statistical treatment of data below the detection limit. Environmental Science and Technology.

Helsel, D.R., and R.J. Gilliom. 1986. Estimation of distributional parameters for censored trace level water quality data: 2. Verification and applications. Water Resources Research. 22:147-155.

Holoway, C.F., J.P. Witherspoon, H.W. Dickson, P.M. Lantz, and T. Wright. 1981. Monitoring for compliance with decommissioning termination survey criteria. Oak Ridge National Laboratory, Oak Ridge, TN. NUREG/CR-2082, ORNL/HASRD-95.

Johnson, R.A., S. Verrill, and D.H. Moore III. 1987. Two-sample rank tests for detecting changes that occur in a small proportion of the treated population. Biometrics. 43:641-655.

Land, C.E. 1971. Confidence intervals for linear functions of the normal mean and variance. Ann. Math. Stat. 42:1187-1205.

Land, C.E. 1975. Tables of confidence limits for linear functions of the normal mean and variance. In: Selected tables in mathematical statistics, Volume III. American Mathematical Society. Providence, RI. pp. 385-419.

McBratney, A.B., and R. Webster. 1981. The design of optimal sampling schemes for local estimation and mapping of regionalized variables. Part 2. Program and examples. Computers and Geosciences 7:335-365.

McBratney, A.B., R. Webster, and T.M. Burgess. 1981. The design of optimal sampling schemes for local estimation and mapping of regionalized variables. Part 1. Theory and method. Computers and Geosciences 7:331-334.

Montgomery, R.H, J.C. Loftis, and J. Harris. 1987. Statistical characteristics of groundwater quality variables. Ground Water, 25(2):176-184.

Nelson, J.D., and R.C. Ward. 1981. Statistical considerations and sampling techniques for groundwater quality monitoring. Ground Water 19:617-625.

Neptune, D., E.P. Brantly, M.J. Messner, and D.I. Michael. 1990. Quantitative decision-making in Superfund: A data quality objectives case study. Haz. Mat. Cont. May/June:19-27.

Neter, J., and W. Wasserman. 1974. Applied Linear Statistical Models. Richard D. Irwin, Inc. Chicago, IL.

Owen, D.B. 1962. Handbook of statistical tables. Addison-Wesley. Reading, MS.

Parkhurst, D.F. 1984. Optimal sampling geometry for hazardous waste sites. Env. Sci. Tech. 18:521-523.

Parkin, T.B., J.J. Meisinger, S.T. Chester, J.L. Starr, and J.A. Robinson. 1988. Evaluation of statistical estimation methods for lognormally distributed variables. Soil Sci. Soc. Am. J. 52:323-329.

Parkin, T.B., S.T. Chester, and J.A. Robinson. 1990. Calculating confidence intervals for the mean of a lognormally distributed variable. Soil Sci. Soc. Am. J. 54:321-326.

Pearson, E.S., and H.O. Hartley. 1966. Biometrika Tables for Statisticians, vol. 1, 3rd ed. Cambridge University Press, London.

Rohde, C.A. 1976. Composite sampling. Biometrics 32: 273-282.

Ryti, R.T., and D. Neptune. 1991. Planning issues for superfund site remediation. Haz. Mat. Cont. Nov/Dec:47-53.

Schaeffer, D.J., and D.G. Janardan. 1978. Theoretical comparison of grab and composite sampling programs. Biom. J. 20:215-227.

Shapiro, S.S., and M.B. Wilk. 1965. An analysis of variance test for normality (complete samples). Biometrika. 52:591-611.

Shapiro, S.S. 1980. How to test normality and other distributional assumptions. In: The ASQC basic references in quality control: Statistical techniques. Volume 3. E.J. Dudewicz (ed.). American Society of Quality Control. Milwaukee, WI.

Sophocleous, M., J.E. Paschetto, and R.A. Olea. 1982. Groundwater network design for northwest Kansas, using the theory of regionalized variables. Ground Water 20:48-58.

U.S. EPA. 1983. Preparation of soil sampling protocol: Techniques and strategies. EPA 600/4-83/020. U.S. Environmental Protection Agency Environmental Monitoring Systems Laboratory. Las Vegas, NV.

U.S. EPA. 1984. Soil sampling quality assurance. EPA 600/4-83/043. U.S. Environmental Protection Agency Environmental Monitoring Systems Laboratory. Las Vegas, NV.

U.S. EPA. 1986. RCRA Groundwater monitoring technical enforcement guidance document (TEGD). U.S. Environmental Protection Agency Office of Solid Waste and Emergency Response 0058.

U.S. EPA. 1988. Methods for evaluating the attainment of cleanup standards — Volume 2: Groundwater. Draft 2.0. U.S. Environmental Protection Agency.

U.S. EPA. 1989a. Methods for evaluating the attainment of cleanup standards — Volume 1: Soils and solid media. EPA 230/02-89-042. U.S. Environmental Protection Agency Office of Policy, Planning and Evaluation, Washington, DC.

U.S. EPA. 1989b. Soil sampling quality assurance. EPA 600/8-89/046. U.S. Environmental Protection Agency Environmental Monitoring Systems Laboratory. Las Vegas, NV.

Van der Parren, J.L. 1970. Tables for distribution-free confidence limits for the median. Biometrika. 57:613-617.

Walpole, R.E., and R.H. Myers. 1989. Probability and statistics for engineers and scientists. Macmillan Publishing Company, New York, NY. 765 pp.

8. EXAMPLES

EXAMPLE 1

CALCULATION OF ARITHMETIC MEAN

We want to calculate the arithmetic mean, $\bar{x}$, of the following data set:

44	80	101	122
55	85	105	129
68	91	110	133
72	94	115	139
76	97	119	167

1. Calculate the sum of all the values:

   $44 + 55 + ... + 139 + 167 = 2{,}002$

2. The arithmetic mean is the sum divided by the number of samples, n. In this case, n = 20, and

   $\bar{x} = 2{,}002/20 = 100.1.$

EXAMPLE 2

CALCULATION OF GEOMETRIC MEAN

We want to calculate the geometric mean of the data set in Example 1.

1. Transform the data by taking the natural logarithm (base e) of each value. The log transformed data are listed below:

3.78	4.38	4.62	4.80
4.01	4.44	4.65	4.86
4.22	4.51	4.70	4.89
4.28	4.54	4.74	4.93
4.33	4.57	4.78	5.12

2. Calculate the sum of the log-transformed data values

   $3.78 + 4.01 + ... + 4.93 + 5.12 = 91.15.$

3. Calculate the arithmetic mean of the log-transformed values (the sum divided by the number of samples, n). In this case, n = 20, so the arithmetic mean of the transformed values is

   $91.15/20 = 4.558.$

4. The geometric mean is the exponent of the mean calculated in Step 3.

   $\exp(4.558) = e^{4.558} = 95.4.$

In this case, the geometric mean is relatively close to the arithmetic mean calculated in Example 1. This is because the data were derived from a normally distributed population. If the data were significantly skewed, the geometric mean would be substantially different from the arithmetic mean.

EXAMPLE 3

METHOD FOR CALCULATING THE MEDIAN OF A DATA SET

Suppose we want to estimate the median of the data set from Example 1.

1. The 20 data are sorted from smallest to largest, and a rank is assigned to each value.

Data	Rank
44	1
55	2
68	3
72	4
76	5
80	6
85	7
91	8
94	9
97	10
101	11
105	12
110	13
115	14
119	15
122	16
129	17
133	18
139	19
167	20

2. Because the sample size, n, is even, the sample median estimate is the average of the n/2th and the (n+2)/2th values. In this case the sample size is 20, the sample median estimate is the average of the 20/2 = 10th and the [(20+2)/2] = 11th ranked values.

3. For this data set, the 10th ranked value is 97 and the 11th ranked value is 101. The median is the arithmetic average of these two points:
   (97 + 101)/2 = 99.

EXAMPLE 4

ESTIMATING A PERCENTILE OF A DATA SET FROM A PROBABILITY PLOT

Twenty soil samples from a site are analyzed for lead, and the following concentrations (ppb) are obtained:

276	179	138	162
206	114	220	131
242	136	157	180
157	165	226	245
146	183	201	193

1. We want to estimate the 50th percentile (median) and 90th percentile of the data set using a probability plot. Assume the data set has been tested for lognormality and normality, and it appears that the data have been drawn from a normal distribution. The 20 data are sorted from smallest to largest, and a rank is assigned to each value. In addition, for each data point estimate (i - 0.5)100/n, where i is the rank of the data point.

Data	Rank	(i - 0.5)100/n	Data	Rank	(i - 0.5)100/n
114	1	2.5	180	11	52.5
131	2	7.5	183	12	57.5
136	3	12.5	193	13	62.5
138	4	17.5	201	14	67.5
146	5	22.5	206	15	72.5
157	6	27.5	220	16	77.5
157	7	32.5	226	17	82.5
162	8	37.5	242	18	87.5
165	9	42.5	245	19	92.5
179	10	47.5	276	20	97.5

2. Because in this case we assume that data are normally distributed, we plot x vs. (i - 0.5)100/n on normal probability paper (contained in this document), as shown below. A straight line is fit to the data by eye, which fits the data reasonably well, indicating that the data are drawn from a normal population.

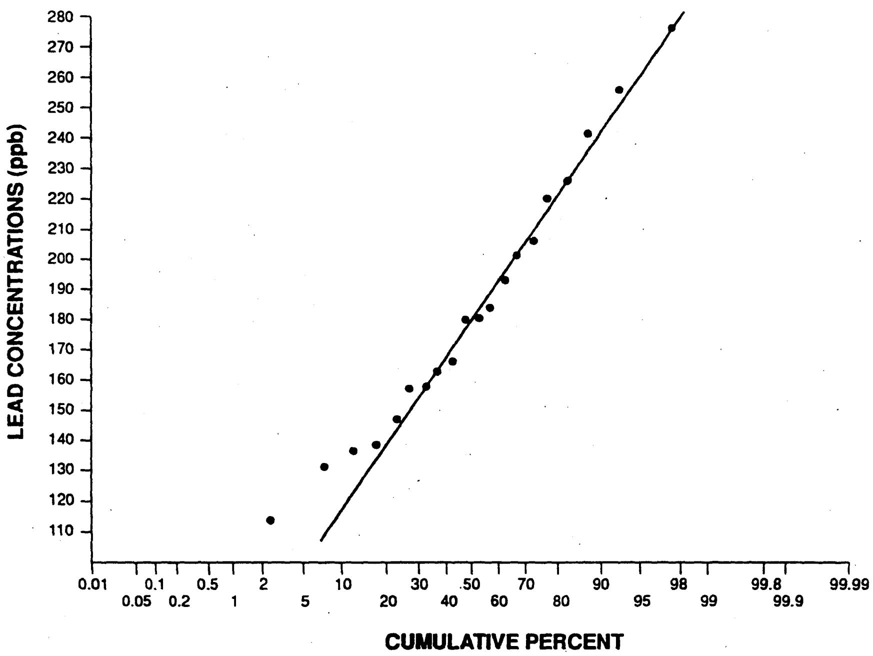


Figure E4. Example of a probability plot.

C704-1909 0192

3. Using the line, we can read the 50th and 90th percentiles from the plot, by reading across the cumulative percent along the horizontal axis to 50 and 90. Using this technique, the 50th and 90th percentile are estimated to be approximately 179 ppb, and 242 ppb. This agrees reasonably well with the median of 179.5 estimated by the method shown in Example 3. A nonparametric method for estimating the 90th percentile is shown in Example 5.

Note: If the data set contained some data below the detection limit or PQL, the data above the limit could be plotted, and a line fit to the remaining data points to estimate upper percentiles.

EXAMPLE 5

NONPARAMETRIC (DISTRIBUTION-FREE) METHOD FOR CALCULATING PERCENTILE OF A DATA SET (Section 2.1.2.3)

Using a nonparametric method, we wish to estimate the 90th percentile of the lead concentration data set in Example 4.

1. The 20 data are sorted from smallest to largest, and a rank is assigned to each value.

Data	Rank	Data	Rank
114	1	180	11
131	2	183	12
136	3	193	13
138	4	201	14
146	5	206	15
157	6	220	16
157	7	226	17
162	8	242	18
165	9	245	19
179	10	276	20

2. $$v = \frac{p}{100}(n + 1)$$

where

p = percentile
n = number of samples
v = rank of pth percentile data

$$v = \frac{90}{100}(20 + 1) = 18.9$$

3. Since v is not an integer, the 90th percentile must be found by linear interpolation between the 18th and 19th ranked data, 242 and 245, respectively.

4. The linear interpolation is performed as follows:

   a. The difference between the rank values is calculated: 19 - 18 = 1
   b. The difference between v and the lower rank value is calculated: 18.9 - 18 = 0.9
   c. The ratio between the values calculated in steps a and b is found: 0.9/1 = 0.9
   d. The difference between the data values is calculated: 245 - 242 = 3.
   e. The ratio in c is multiplied by the difference between the data values: 0.9 (3) = 2.7
   f. This value is added to the lowest data value: 242 + 2.7 = 244.7.

Thus, the 90th percentile of the data set is 244.7.

EXAMPLE 6

CALCULATION OF VARIANCE, STANDARD DEVIATION, AND COEFFICIENT OF VARIATION

We want to calculate the sample variance, s^2, of the following concentrations (x_i) in mg/kg : 2.4, 4.4, 6.5, 6.7, and 8.2.

The arithmetic mean, $\bar{x}$, calculated as described in Example 1, is 5.64. The equation for calculating the variance is

$$s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n - 1}$$

where n is the number of samples. In this example, n = 5, so the denominator is 5 - 1 = 4. Thus, the sample variance can be calculated as

$$s^2 = \frac{(2.4 - 5.64)^2 + (4.4 - 5.64)^2 + (6.5 - 5.64)^2 + (6.7 - 5.64)^2 + (8.2 - 5.64)^2}{4}$$

$$s^2 = 5.11$$

The standard deviation, s, is the square root of the variance:

$$s = \sqrt{5.11} = 2.26.$$

The coefficient of variation, CV, is calculated by:

$$CV = \frac{s}{\bar{x}}$$

$$CV = \frac{2.26}{5.64} = .040$$

NOTE: This calculation method should not be used for some purposes. See Supplement S-5.

EXAMPLE 7

W TEST FOR TESTING THE NORMALITY OF A DATA SET

The data from the 20 soil samples in Example 4 will be tested for normality by the W test (Shapiro and Wilk 1965). These data could be tested for lognormality by log-transforming the date, and then performing the same test on the transformed data.

1. The number of samples, n, is 20. The calculated mean, $\bar{x}$, of the data is 182.85.
2. The 20 data are sorted from smallest to largest, and a rank is assigned to each value

Data	Rank	Data	Rank
114	1	180	11
131	2	183	12
136	3	193	13
138	4	201	14
146	5	206	15
157	6	220	16
157	7	226	17
162	8	242	18
165	9	245	19
179	10	276	20

3. The denominator d is calculated for the data:

$$d = (114 - 182.85)^2 + (131 - 182.85)^2 + (136 - 182.85)^2 + (138 - 182.85)^2 + (146 - 182.85)^2 + (157 - 182.85)^2 + (157 - 182.85)^2 + (162 - 182.85)^2 + (165 - 182.85)^2 + (179 - 182.85)^2 + (180 - 182.85)^2 + (183 - 182.85)^2 + (193 - 182.85)^2 + (201 - 182.85)^2 + (206 - 182.85)^2 + (220 - 182.85)^2 + (226 - 182.85)^2 + (242 - 182.85)^2 + (245 - 182.85)^2 + (276 - 182.85)^2$$

$$d = 35{,}355$$

4. Calculate r, the number of a_r coefficients used in the calculation.
   Since n is even, $r = 20/2 = 10$

5. From Table A-1, the a_r coefficients for $n = 20$ are:

$a_1 = 0.4734$	$a_6 = 0.1334$
$a_2 = 0.3211$	$a_7 = 0.1013$
$a_3 = 0.2565$	$a_8 = 0.0711$
$a_4 = 0.2085$	$a_9 = 0.0422$
$a_5 = 0.1686$	$a_{10} = 0.0140$

6. W is calculated as follows:

$$W = (1/35{,}355)\,[0.4734(276 - 114) + 0.3211(245 - 131) + 0.2565(242 - 136)] + 0.2085(226 - 138) + 0.1686(220 - 146) + 0.1334(206 - 157) + 0.1013(201 - 157) + 0.0711(193 - 162) + 0.0422(183 - 165) + 0.0140(180 - 179)]^2$$

$$W = 0.97$$

Example 7. (Continued)

---

7. Using Table A-2, the value of W for the significance level $\alpha = 0.05$ is 0.905. The value for W calculated in Step 6 above (0.97) is greater than the value in Table A-2, so the null hypothesis that the population is normally distributed cannot be rejected, and the data should be assumed to have been drawn from a normal distribution.

NOTE: If calculating W by hand, use Worksheet W-1 or W-1a.

EXAMPLE 8

TRANSFORMATION OF LOGNORMALLY DISTRIBUTED DATA

The following groundwater concentrations of a contaminant (mg/l) have been measured at a site:

82	151	75	105
61	68	100	123
95	74	126	85
136	163	112	89
59	99	108	88

1. A histogram of the data suggests that they may be lognormally distributed (Figure 5a).

2. To test the lognormal assumption, the data are log-transformed ($\log_e$).

4.41	5.02	4.32	4.65
4.11	4.22	4.61	4.81
4.55	4.30	4.84	4.44
4.91	5.09	4.72	4.49
4.08	4.60	4.68	4.48

3. A histogram of the transformed values (Figure 5b) indicates that they are normally distributed. This suggests that the data are lognormally distributed. Note that the histogram method for determining normality or lognormality is subjective and depends on the intervals chosen for the graph. It is used here to illustrate lognormal transformations. In practice, a probability plot or other test (e.g., W test) should be used to determine normality or lognormality (see Supplement S-3).

EXAMPLE 9

PARAMETRIC AND NONPARAMETRIC METHODS DETERMINING WHETHER A CLEANUP STANDARD IS BELOW NATURAL BACKGROUND – NORMALLY DISTRIBUTED DATA

The cleanup standard for kryptonite in soil is 115 mg/kg, based on Method B of the MTCA regulations. The PLP collects 20 samples from locations determined to be natural background. Is the Method B cleanup standard below natural background?

The background data are:

110.28	107.11	61.56	91.81	89.08
116.32	20.06	112.84	101.87	64.52
124.14	91.80	50.28	97.04	91.55
111.94	78.54	110.17	80.78	155.19

Background data are assumed initially to be lognormally distributed (see the discussion in Section 4.3 and the flowchart for determination of cleanup standards based on background, Figure 12). To check the lognormal assumption for the kryptonite background data, the W test (see Example 7) will be used. As discussed in Section 2.1.4.1, the W test is designed as a test of the hypothesis that the data are from a normal distribution. However, it can be used to test the hypothesis of a lognormal distribution by first transforming the raw data using natural logarithms, and then calculating and evaluating the W statistic using the transformed data as shown in Example 7.

The W statistic for the transformed data is calculated to be 0.792. The critical value at the 0.05 level for a sample size of 20 is determined from Table A-2 to be 0.905. Since the calculated value for W is less than the critical value, the null hypothesis that the data are lognormal is rejected. The PLP then decides to determine if the data are normally distributed. The W value calculated on the raw (untransformed) data is 0.958, which exceeds the critical value of 0.905, so the null hypothesis that the data are normally distributed is not rejected.

The default procedure establishing the background-based cleanup standard at the 90th percentile of the estimated distribution is based on the assumption of a lognormal distribution. In cases where the assumption of a lognormal distribution is rejected, as in this example, Ecology should be consulted on appropriate alternative procedures to establish the cleanup standard. The 90th percentile concentration <u>should</u> <u>not</u> be used without consulting Ecology. For this example, assume that Ecology has determined that the cleanup standard should be based on the estimated 80th percentile concentration.

The 80th percentile then can be estimated using a table of standard normal values (see Table A-6) as follows:

$$x_{80} = \bar{x} + Z_{80}s$$

and, since

$$\bar{x} = 93.34$$

$$s = 29.33$$

and from Table A-6

$$Z_{80} = 0.842$$

then

$$x_{80} = 93.34 + 1.282(29.33)$$

$$= 118.04 \text{ mg/kg}$$

This procedure assumes a normal distribution for the background samples to estimate the 80th percentile value. Since that estimate, about 118 mg/kg, is greater than the Method B cleanup standard of 115 mg/kg, the Method B value is below natural background.

Now, assume that we wish to estimate the 80th percentile value of the background distribution using nonparametric methods as in Example 5.

First, the 20 background values are ranked from lowest to highest:

20.06	97.04
50.28	101.87
61.56	107.11
64.52	110.17
78.54	110.28
80.78	111.94
89.08	112.84
91.55	116.32
91.80	124.14
91.81	155.19

Then the 80th percentile is estimated as in Example 5, as follows:

$$k = \frac{p}{100}(n+1) = 0.80(21) = 16.8$$

and interpolating between the 16th and 17th ranked values, 111.94 and 112.84,

$$x_{80} = 111.94 + 0.8(112.84 - 111.94)$$

$$= 111.94 + 0.72$$

$$= 112.66 \text{ mg/kg}$$

Example 9. (Continued)

Since this estimate of the 80th percentile of the background data is less than the 115 mg/kg from Method B, the Method B cleanup standard is not below natural background.

This example demonstrates that the method of estimating the 80th percentile of natural background can affect whether or not a Method A, Method B, or Method C cleanup standard is determined to be below natural background. In this example, the difference in 80th percentile values from parametric and nonparametric approaches is not great; in other cases it may be much greater, and either approach can produce the higher estimated 90th percentile value. In this case, because the background-based cleanup standard is so close to the Method B standard, and the parametric and nonparametric methods result in different decisions, it would be wise to collect more background samples.

In general, the first (parametric) method shown here should be used unless the data deviate significantly from normal and lognormal distributions. If the data are lognormally distributed, as assumed in the default procedures and typically expected for most environmental data, see Example 10. Consult Ecology before using the nonparametric method.

EXAMPLE 10

PARAMETRIC AND NONPARAMETRIC METHODS DETERMINING WHETHER A CLEANUP STANDARD IS BELOW NATURAL BACKGROUND – LOGNORMALLY DISTRIBUTED DATA

The cleanup standard for ubiquinite in soil is 175 mg/kg, based on Method B of the regulations. The PLP collects 20 samples from locations determined to be natural background. Is the Method B cleanup standard below natural background?

The background data (rank-ordered) are:

42	78	95	122
61	81	98	132
66	83	104	138
69	85	109	212
71	90	114	286

The assumption that the background data are from a lognormal distribution is evaluated using the W test (see Example 7). The W test is used to evaluate a null hypothesis that background values are lognormally distributed by transforming the raw data using natural logarithms and then calculating the W statistic, as in Example 9. For the log-transformed ubiquinite data listed above, the W statistic is calculated to be 0.953; since that value is greater than the critical value of 0.905 (at the 0.05 level) from Table A-2, the null hypothesis is not rejected, and a lognormal distribution is assumed.

If the lognormal null hypothesis is not rejected, it is not necessary to test other distributions.

The 90th percentile of the lognormally-distributed background data can be estimated by transforming the raw data using logarithms, estimating a 90th percentile value using a table of standard normal values (see Table A-6), and then back-transforming to original units (appropriate for percentiles, but not for means!), as follows:

Transform the raw data using logarithms, with $y_i = \log_e x_i$:

3.738	4.357	4.554	4.804
4.111	4.394	4.585	4.883
4.190	4.419	4.644	4.927
4.234	4.443	4.691	5.357
4.263	4.500	4.736	5.656

The 90th percentile of the transformed (normal) data can be estimated as in Example 9:

$$y_{90} = \bar{y} + Z_{90} s_y$$

The mean and standard deviation of the transformed data are:

$$y = 4.574$$

$$s_y = 0.429$$

and from Table A-6

$$Z_{90} = 1.282.$$

Then

$$y_{90} = 4.574 + 1.282(0.429)$$
$$= 5.124$$

Finally, transform back to the original units:

$$x_{90} = e^{5.124} = 168.0$$

This procedure uses the assumed lognormal distribution for the background samples to estimate the 90th percentile value. Since that estimate, 168.0 mg/kg, is less than the Method B cleanup standard of 175 mg/kg, the Method B value is determined not to be below natural background.

Now assume that we wish to estimate the 90th percentile value of the nontransformed background distribution using nonparametric methods, as in Example 5.

The 90th percentile is estimated as in Example 5, as follows:

$$k = p/100\ (n+1) = 90/100\ (21) = 18.9$$

and interpolating between the 18th and 19th ranked values, 138 and 212,

$$x_{90} = 138 + 0.9(212 - 138)$$
$$= 138 + 66.6$$
$$= 204.6$$

Since this estimate of the 90th percentile of the background data is greater than the 175 mg/kg from method B, the Method B cleanup standard is determined to be below natural background.

As in Example 9, this example demonstrates that the method of estimating the 90th percentile of natural background can affect whether or not a Method A, Method B, or Method C cleanup standard is determined to be below natural background. In this example, the nonparametric estimate of $x_{0.90}$ is substantially higher than the parametric estimate. The first (parametric) method shown here should be used unless the data deviate significantly from normal and lognormal distributions. Consult Ecology before using the nonparametric method.

EXAMPLE 11

EVALUATION OF SOILS COMPLIANCE MONITORING DATA

The Method B cleanup standard for a contaminant, Pesticite, in soils is 2.5 μg/kg. Assume that Pesticite is widely distributed in areas near the site (a pesticide distribution facility) from normal agricultural practices, unrelated to any site-specific releases. Appropriate and representative area background samples are collected and analyzed. It is determined from those samples that the assumption of a lognormal area background distribution is not rejected (at the 0.05 level) and that the estimated 90th percentile concentration of area background is 30 μg/kg. Then the Method B cleanup standard is less than area background, and a Method C standard equal to the 90th percentile of area background, 30 μg/kg, is selected in accordance with the default procedures discussed in section 4.3.

A compliance monitoring data set of 15 samples is collected and analyzed for Pesticite after site remediation activities are performed. Can the site be considered clean?

Assume the monitoring data are as follows:

ND (<5)	16	25
5	19	29
6	21	30
8	21	32
12	24	38

The W test (see Example 7) is used to evaluate the null hypothesis that the compliance monitoring data are from a lognormal distribution. The W statistic is calculated assuming a value of 2.5 μg/kg (one-half the detection limit) for the one not-detected (ND) value. The goodness-of-fit test for a lognormal distribution is performed by transforming the raw data using natural logarithms ($\log_e$), as in Example 10. The calculated W statistic is 0.888, which is slightly greater than the critical value of 0.881 for a 0.05 test with 15 samples (see Table A-2). Therefore, the null hypothesis of a lognormal distribution is not rejected.

Although the W test cannot reject a lognormal distribution, a histogram plot of the compliance monitoring data reveals that, subjectively, the lognormal distribution does not provide a close fit to the data. The compliance monitoring data appear to be a combination of lognormally-distributed background data (approximating the area background distribution) and a second distribution shifted upward to higher concentrations compared to that background distribution. The higher values in the compliance monitoring data set all appear to be from samples taken in an area of the site known to have been used for storing bags of dry Pesticite mix. That area was the most contaminated at the site before cleanup actions were taken.

Three criteria will be used to evaluate whether the site is in compliance:

1) Calculation of the upper confidence limit (UCL) on the mean, and comparison of that value to the 90th percentile of the background data

2) Frequency of exceedance

3) Magnitude of exceedances.

Example 11. (Continued)

CRITERION 1

Since the null hypothesis of a lognormal distribution is not rejected, the method of Land (1971, 1975) described in Gilbert (1987) is used to calculate a one-sided 95 percent upper confidence limit on the mean concentration of Pesticite in soil at the site. The equation for that upper confidence limit (UCL) is as follows:

$$UCL_{95} = \exp(\bar{y} + 0.5s_y^2 + \frac{s_y H_{95}}{\sqrt{n-1}})$$

where

$\bar{y}$ = arithmetic mean of the n transformed values $y_i = \ln x_i$

s_y = standard deviation of the transformed data

n = the number of sampled values

H_{95} = tabled values from Land (1971, 1975) determined by n and s_y

The raw data are transformed using logarithms ($\log_e$):

0.916	2.773	3.219
1.609	2.944	3.367
1.792	3.045	3.401
2.079	3.045	3.466
2.485	3.178	3.638

The values for the mean and standard deviation of these transformed data are determined (to three decimal places):

$\bar{y}$ = 2.730

s_y = 0.794

The approximate value of H can be determined from the nomograph in Figure A-1 or Supplement S-2. In this case, n = 15 samples, and s_y = 0.794. Interpolating between 2.306 and 2.443, the H value is determined to be 2.434. The UCL on the mean is then calculated using these values, as follows:

UCL_{95} = exp(2.730 + 0.5(0.630) + 0.794(2.434)(1/[14]$^{0.5}$))

= exp(3.562)

UCL_{95} = 35.2 ug/kg

Example 11. (Continued)

The calculated UCL of 35.2 μg/kg, from the compliance monitoring data, is therefore greater than the cleanup standard of 30 μg/kg based on the 90th percentile of area background, and the site does not meet the applicable cleanup standard based on background. Further cleanup actions are required. In this case, the residual soil contamination located where dry Pesticite mix was stored appears to cause the site to fail the test.

The exceedance of the background-based cleanup standard based on this test is sufficient by itself to require additional cleanup actions. Two additional tests are included in the regulation to account for the frequency and magnitude of exceedances of the background-based cleanup standard in the compliance monitoring data. For purposes of illustration, these tests are also discussed here, although once the test based on the UCL of the mean is failed, they would not necessarily have to be performed.

CRITERION 2

According to the regulation, the standard test, based on the frequency of exceedances of the cleanup level, is that no more than 10 percent of the compliance monitoring samples exceed the cleanup level. However, the actual probability of having more than 10 percent of compliance monitoring samples above the cleanup level if the site is at background concentrations is rather high, and generally increases as the compliance monitoring sample size increases (see Technical Attachment 1 to Figure 12). For sample sizes of 20 or more and a cleanup level based on the 90th percentile concentration of background, that probability is greater than 0.30. This is not acceptable, because even if a site is remediated to background concentrations (obviously a "clean site"), it has a >30 percent chance of failing this test. Therefore, an adjustment in the allowable percentage of compliance monitoring samples above the cleanup level can be made so that the "false positive" error rate approximates 5 percent. This adjustment is made <u>only</u> in the case of a cleanup level based on background.

For sample sizes less than or equal to 30, an appropriate adjustment is to allow up to 20 percent of the samples to exceed the cleanup level (see Attachment 1). Therefore, for the Pesticite site with 15 samples, the test based on frequency of exceedances would require additional cleanup actions if 4 or more out of 15 compliance monitoring samples exceeded the cleanup level. The calculated probability of 4 or more exceedances if the site has achieved background is 0.056 (5.6 percent). For the compliance monitoring data reported at the Pesticite site, 2 of the 15 values exceed the cleanup level of 30 μg/kg. The exceedance frequency is therefore 13.3 percent. Although this is greater than the standard test criterion of 10 percent, it is less than the adjusted criterion (for the specific background-based cleanup standard and compliance monitoring sample size) of 20 percent. The site does not fail the test of frequency of exceedances.

CRITERION 3

The regulation states that no compliance monitoring sample be more than two times the cleanup level. The probability of one or more samples exceeding two times the cleanup standard if the site is at background concentrations depends on the definition of the cleanup standard (i.e., the percentile value selected as the cleanup level), the shape of the background distribution (e.g., the coefficient of variation [CV], defined as the standard deviation divided by the mean concentration), and the number of compliance monitoring samples. A factor of exceedance that results in an approximate 5 percent false positive rate can be calculated (see Technical Attachment 2 to Figure 12).

Example 11. (Continued)

For the Pesticite site, assume that the background data are lognormally distributed with a CV of 0.7. The probability of exceeding twice the cleanup level (the 90th percentile concentration) in 15 compliance monitoring samples (false positive probability) is about 0.123 (12.3 percent). However, for a more acceptable rate of exceedence probability (0.05, or 5 percent), a factor of about 2.46 is calculated (see Technical Attachment 2 to Figure 12). The exceedance factor of 2.46 means that a sample may exceed the cleanup standard by up to 2.46x, instead of 2. Such an adjustment in the standard criterion can be made only in the case of a cleanup level based on background.

For the compliance monitoring data reported at the Pesticite site, the maximum concentration of 38 µg/kg is only 1.27 times the cleanup level of 30 ug/kg. That exceedance factor is less than the adjusted criterion value of 2.46 for the site. Therefore, the Pesticite site does not fail the magnitude of exceedence (Criterion 3) test.

The compliance monitoring data for this site strongly suggest a residual hot spot of contamination (e.g., based on the histogram of soil concentrations and their spatial pattern at the site). Alternative statistical procedures based on distributional tests such as the Wilcoxon or Quantile tests may be appropriate and useful in such situations (see Figure 12).

EXAMPLE 12

DETERMINATION OF GROUNDWATER CLEANUP STANDARDS BASED ON NATURAL BACKGROUND DATA

The Zarkle Industries site, now closed, is contaminated with the inorganic constituent Zodium, which was used in large quantities in a manufacturing process. In particular, groundwater concentrations of Zodium at the site are very high (mg/l levels). The Method B Cleanup standard for Zodium in groundwater is 0.5 μg/l, based on acceptable human health risks (drinking water ingestion). Is the Method B cleanup standard below natural background?

Several articles in the literature have noted that natural background concentrations of Zodium in groundwater are quite variable. The PLP decides to drill monitoring wells to determine background concentrations near the Zarkle Industries site. Sampling locations are selected carefully and reviewed with Ecology to screen out any locations that could be influenced by site contamination. The background results for Zodium (in μg/l) are as follows:

9.74	22.39
14.74	1.98
2.20	2.31
27.39	0.56
0.86	75.07

The higher results are reviewed by Ecology and the PLP to confirm that they represent background values and are not influenced by the site or other identifiable sources. No reason for rejecting the higher results is found; moreover, these higher concentrations are consistent with previous literature reports on Zodium. The results are accepted for determining natural background.

The assumption that the background data are from a lognormal distribution is evaluated using the W test (see Example 7). As in Examples 9 and 10, the W test is used to evaluate a null hypothesis that background values are lognormally distributed by transforming the raw data using natural logarithms and then calculating the W statistic. The calculated W statistic of 0.945 is compared to the critical value of 0.842 based on 10 samples and a 0.05 level test (Table A-2). Since the calculated W statistic is greater than the criterion value, the null hypothesis of a lognormal distribution is not rejected, and the data are assumed to be lognormally distributed.

The estimated 90th percentile background concentration is calculated as in Example 10. The $\log_e$ transformed data are as follows (to three decimal places only):

2.276	3.109
2.691	0.683
0.788	0.837
3.310	-0.580
-0.151	4.318

The mean, $\bar{y}$, and standard deviation, s, of the log-transformed data are 1.728 and 1.632, respectively.

The 90th percentile of these log-transformed data is obtained by finding the 90th percentile value based on the best-fit transformed normal distribution and then back-transforming (appropriate for percentiles, but not for means!), as follows:

Example 12. (Continued)

$$y_{90} = \bar{y} + z_{90}s_y$$

$$= 1.728 + 1.282(1.632)$$

$$= 3.820$$

Transforming back to original units, the estimated 90th percentile of the natural background for Zodium in groundwater is

$$x_{90} = e^{3.820} = 45.60\ \mu g/l$$

Therefore, the Method B cleanup standard of 0.5 μg/l for Zodium is below natural background. Before accepting the estimated 90th percentile of the natural background distribution for Zodium as the cleanup standard, however, the distribution of background values is considered further. The estimated 90th percentile value of 45.60 μg/l is more than 91 times higher than the risk-derived Method B standard of 0.5 μg/l.

The background data set shows considerable positive skew. The coefficient of variation (CV) of the best-fit lognormal distribution is calculated as the standard deviation divided by mean concentration, or about 3.65. Upper percentile values for distributions with that degree of skew are well above the typical values around the 50th percentile of the distribution.

The 50th percentile value is easily determined. Recall that the transformed data from a lognormal distribution are normally distributed, and the mean and 50th percentile (median) values for a normal distribution are identical. The 50th percentile value is therefore calculated using the mean of the log-transformed background data as follows:

$$y_{50} = 1.728$$

$$x_{50} = e^{1.728} = 5.63\ \mu g/L$$

The 90th percentile value of 45.60 μg/L is therefore about 8.1 times higher than the 50th percentile value. Thus, the calculated risks at the 90th percentile are also more than 8 times higher than at more typical background concentrations. Ecology policy is to limit the cleanup standards based on natural background to no more than 4 times the 50th percentile background concentration. This policy represents a balancing of acceptable exposures and risks with the probability that clean (background) sites would fail a comparison with the cleanup standard for a site.

The cleanup standard for Zodium in groundwater at the Zarkle Industries site based on natural background is therefore 4 times the 50th percentile value, or

$$\text{Cleanup Standard} = (4)\ (5.63)$$

$$= 22.52\ \mu g/L$$

Example 12. (Continued)

This cleanup standard represents a value only slightly greater than the 80th percentile of the background distribution. The estimated percentile can be calculated by determining the Z value of a normal distribution that solves the equation

$$\log_e 22.52 = \bar{y} + Z_p s_y$$

and then looking up that Z value in Table A-6 to determine the percentile.

The effect for this site of limiting the natural background cleanup standard to 4 times the 50th percentile value is to adopt the 80th rather than the 90th percentile of the background distribution. This results in a higher probability that a clean (background) site would fail the test (higher false positive rate), balanced by almost a 50 percent reduction in the exposures that would occur at the estimated 90th percentile of natural background.

The percentile value for the cleanup standard of 22.52 μg/l is estimated as 80.22. A test based on the frequency of values above the cleanup standard at that percentile can be derived for a given compliance monitoring sample size to provide an approximate 5 percent false positive rate (i.e., an approximate 0.05 level test). Using the approach described in Technical Attachment 1 of Figure 12, for example, the probability that 5 or more out of 10 compliance monitoring samples would exceed a cleanup standard based on the 80.22nd percentile value is 0.031. Therefore, a test at the 0.03 level for 10 samples would be based on not more than 40 percent of the values exceeding the cleanup level.

A test based on the maximum magnitude of exceedance can be derived similarly using the approach described in Technical Attachment 2 of Figure 12. For example, with a compliance monitoring sample size of 10, there is a 5-percent chance that one or more values would exceed the 99.49th percentile if the site is at background. For a lognormal distribution with a CV of about 3.65, as estimated for the Zodium background distribution in groundwater, a test at the 0.05 level of the maximum magnitude observed would be based on an exceedance factor of about 16.5 times the cleanup standard.

Background data sets with high coefficients of variation (highly skewed distributions) will pose problems for simultaneously achieving desirable false positive error rates and statistical power to detect residual contamination, using the standard default methods described in this guidance document and based on relatively small numbers of samples. In such cases, it may be appropriate to consider using alternative distribution testing methods such as the Wilcoxon and Quantile tests. In the context of the standard default procedures, values of the lognormal distribution CV (the mean divided by the standard deviation of the best-fit lognormal distribution, not the raw sample values) greater than about 0.5 may be considered relatively high.

EXAMPLE 13

CONFIDENCE INTERVAL METHOD FOR TESTING COMPLIANCE - NORMALLY DISTRIBUTED DATA

We wish to evaluate the soil lead data from Example 4 to determine whether the site complies with the risk-based 250-mg/kg cleanup level (Method A).

276	179	138	162
206	114	220	131
242	136	157	180
157	165	226	245
146	183	201	193

n	20
$\bar{x}$	182.85
s	43.14

Assume that these data have been tested for lognormality and normality and are assumed to be normally distributed. Find the appropriate t-value in a t table (Appendix Table A-4). There are n − 1 = 19 degrees of freedom, and the correct column is for 0.05. The t-value is therefore 1.729. Calculate the upper confidence limit (UCL):

$$UCL = 182.85 + 1.729 \frac{43.1}{\sqrt{20}}$$
$$= 199.5.$$

Since the UCL on the mean is less than 250, the site meets the criteria that the UCL must be less than the cleanup standard. In addition, no single value is greater than two times the cleanup standard, and fewer than 10 percent (5 percent) of the values are above the cleanup standard. Therefore, the site can be considered uncontaminated. As described in Section 5.2.1, the confidence interval method should be used to compare compliance monitoring data to cleanup levels based on chronic or carcinogenic effects.

EXAMPLE 14

TOLERANCE INTERVAL METHOD FOR TESTING COMPLIANCE

Use the same data as for Example 13. From Table A-3 under $P_0 = 0.1$ for $n = 20$ (sample size), $k = 1.926$. The value for P_0 comes from the fact that this is a confidence interval around the 90th percentile ($100 - 90 = 10$; $\frac{P_o}{100} = 0.10$). Then:

$$T_U = 182.85 + (1.926)(43.14)$$
$$= 265.9$$

Although as shown in Example 13, the site meets two requirements for it to be considered clean, because the cleanup level of 250 falls below the upper tolerance interval, the site is not clean by this test. In general, the tolerance interval approach will be more stringent than the confidence interval method. As described in Section 5.2.2, the tolerance interval method should be used to compare compliance monitoring data to cleanup levels based on acute or short-term effects.

EXAMPLE 15

CALCULATION OF NONPARAMETRIC CONFIDENCE LIMITS FOR PERCENTILES WHEN n ≤20 (Section 5.2.2.3)

Assume that the following soils data set (in mg/kg) from a site has been tested for normality and lognormality, and found to fit neither distribution. The cleanup standard is 7 mg/kg. Does the site meet the cleanup standard for soils?

6.01	4.53	6.78	3.79
8.94	8.20	5.43	6.54
5.21	4.46	5.90	6.23
8.43	5.32	7.42	4.01

We want to find the upper 95 percent confidence limit around the 90th percentile for the data set.

A nonparametric test must be applied because the data are assumed not to be normally or lognormally distributed. In addition, n≤20, so the procedure described in Conover (1980) should be applied.

1. In this case, $\alpha = 0.05$, $1-\alpha = 0.95$, p/100 = 0.90, and n = 16. Conover provides two-sided confidence intervals in this table. However, under the regulation a one-sided interval is the appropriate test. Therefore, for an equivalent one-sided test at $\alpha = 0.05$, the two-sided test for twice that value, or 0.10 must be used. Table A-5 is used to find values for b. Read across columns for p/100 = 0.90, and down the left hand column to n = 16. Move down the column (p/100 = 0.90) until the values approximate $\alpha/2$ (in this case 0.05). Find the corresponding value of y in the far left column. In this example, a tabled value of 0.0684 is closest to 0.05, and the corresponding y value is 12. This value is r−1; we must add 1 to get r. Thus, in this example, r = 13.

2. Continue down the column for p/100 = .90, until you reach an entry approximately equal to $1-(\alpha/2)$, which is 0.950 in this example. The closest value is 0.8147, with a corresponding y value of 15. Again, this value represents s−1, so 1 is added to obtain to obtain s (16).

3. We wish to find the data values corresponding to ranks of r = 13 and s = 16. Order the data from smallest to largest and assign a rank to each value.

Data	Rank
3.79	1
4.01	2
4.46	3
4.53	4
5.21	5
5.32	6
5.43	7
5.90	8
6.01	9
6.23	10
6.54	11
6.78	12
7.42	13
8.20	14
8.43	15
8.94	16

4. The data values corresponding to ranks of 13 and 16 are 7.42 and 8.94. These values represent the lower and upper confidences limits about the 90th percentile. The upper confidence limit is 8.94 which is greater than the cleanup standard of 7 mg/l. Thus, the site should be considered contaminated.

Technical Note: In this case, the exact confidence coefficient is determined by the α entries used, and is not equal to 0.95. The actual value in this case is [1-0.017-0.8147 = 0.1683], not 0.05.

EXAMPLE 16

CALCULATION OF NONPARAMETRIC CONFIDENCE LIMITS FOR PERCENTILES WHEN N > 20 (Section 5.2.2.4)

Suppose we want to calculate the upper 95-percent confidence limit around the 90th percentile for a data set with n = 100.

Assume that these data have been tested and do not appear to be normally or lognormally distributed. Thus, we select the nonparametric method for estimating one-sided upper confidence limits (Gilbert 1987, p.141).

1. Find $Z_0 = Z_{0.95}$ in Table A-6. This value can be found by searching the table for a value close to 0.95. In Table A-6, the closest values are 0.9495 and 0.9505. Z_p is found by reading the left-hand column value (Z_p) in the same row as these values. In this case, the value is 1.6. The hundredths digit must then be interpolated from the values in the upper row. In this case, we must interpolate between 0.04 and 0.05 (corresponding to 0.9495 and 0.9505). (An example of linear interpolation is shown in Example 5.) Thus, $Z_p = 1.6 + 0.045 = 1.645$.

2. Calculate the rank of the upper confidence limit:

$$u = \frac{p}{100}(n+1) + Z_{1-\alpha}\sqrt{(n)(\frac{p}{100})(1 - \frac{p}{100})}$$

$$u = \frac{90}{100}(100+1) + 1.645\sqrt{(100)(\frac{90}{100})(1 - \frac{90}{100})}$$

$$u = 95.835$$

3. Order the data from smallest to largest and assign a rank to each value.

4. Since u is not an integer, the limit must be obtained by linear interpolation between the 95th and 96th ranked data values.

EXAMPLE 17

NONPARAMETRIC METHOD FOR EVALUATING GROUNDWATER COMPLIANCE (Section 5.3.3)

The method of Van der Parren will be used to estimate an upper confidence limit on the median. The following concentrations (mg/l) of XYZ are measured in the groundwater at a site. Does the groundwater meet the cleanup level of 6 ppm?

Raw data	Sorted data	
XYZ (ppm)	Rank	XYZ(ppm)
9.3	1	0.4
5.6	2	1.5
0.4	3	1.6
3.1	4	1.8
2.0	5	1.9
4.2	6	1.9
1.9	7	1.9
7.4	8	2.0
6.1	9	2.3
5.3	10	2.5
1.9	11	3.1
1.8	12	3.5
1.5	13	3.7
3.7	14	4.2
2.5	15	5.3
1.6	16	5.6
5.8	17	5.8
3.5	18	6.1
2.3	19	7.4
1.9	20	9.3

The sample size (n) is 20. From Appendix Table A-7, j=15. For the sorted data, the 15th value (= upper confidence limit) is 5.3 which is less than the cleanup standard of 6 ppm. Therefore, the criterion that the 95 percent confidence limit on the median must be below the cleanup standard is satisfied. However, 15 percent (more than 10 percent) of the data are above the cleanup standard, so this criterion is not met, and the site is still considered to be contaminated.

Technical Note: The true confidence coefficient defined in Van der Parren for this case is 0.9586.

APPENDIX A

Figure A-1. Upper 95% Confidence Limits for a Lognormal Distribution (H values)

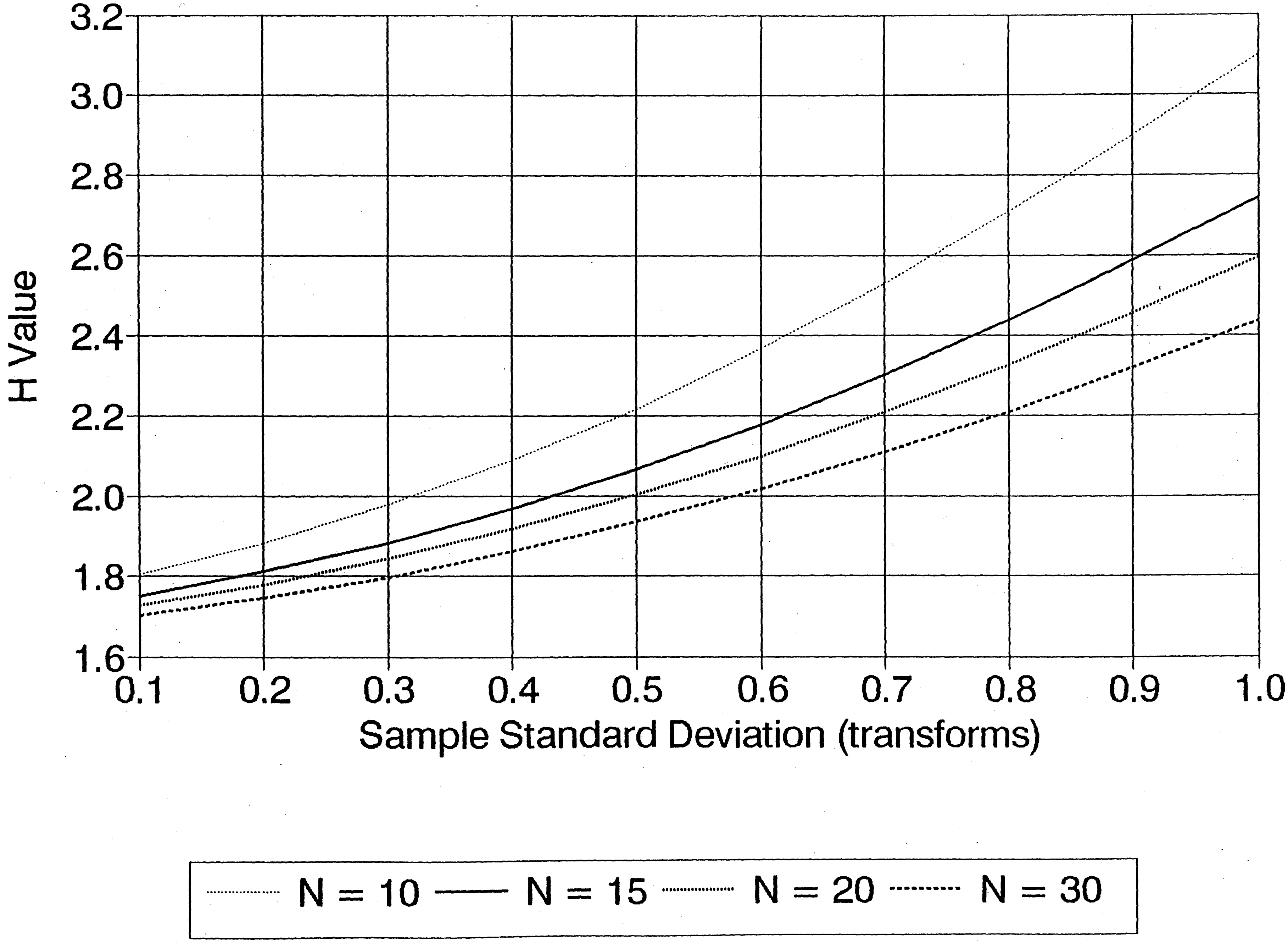


Figure A-1. Upper 95% Confidence Limits for a Lognormal Distribution (H values)

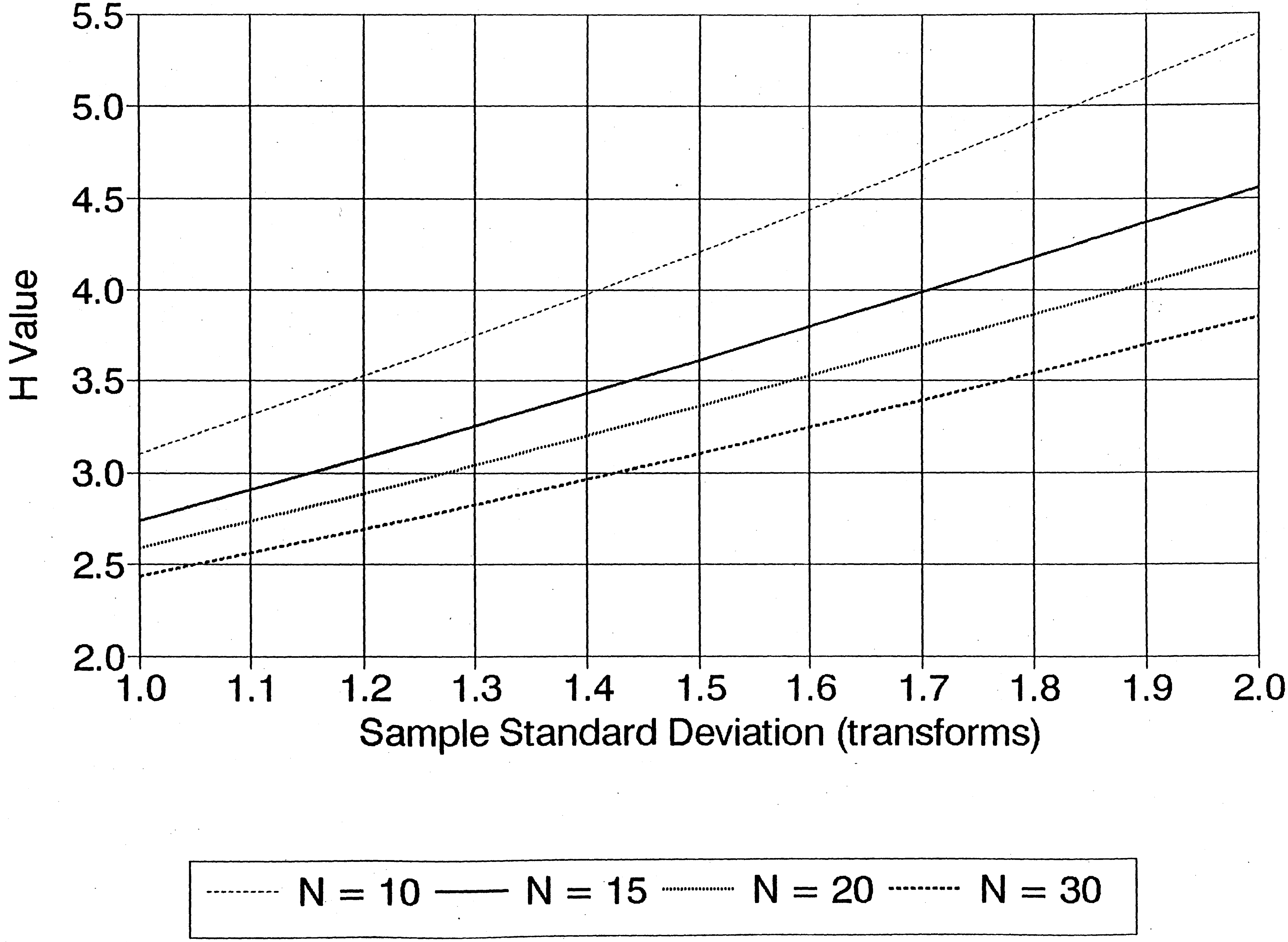


TABLE A-1. COEFFICIENTS a_i FOR THE SHAPIRO-WILK W TEST FOR NORMALITY

i \ n	2	3	4	5	6	7	8	9	10
1	0.7071	0.7071	0.6872	0.6646	0.6431	0.6233	0.6052	0.5888	0.5739
2	-	0.0000	0.1677	0.2413	0.2806	0.3031	0.3164	0.3244	0.3291
3	-	-	-	0.0000	0.0875	0.1401	0.1743	0.1976	0.2141
4	-	-	-	-	-	0.0000	0.0561	0.0947	0.1224
5	-	-	-	-	-	-	-	0.0000	0.0399

i \ n	11	12	13	14	15	16	17	18	19	20
1	0.5601	0.5475	0.5359	0.5251	0.5150	0.5056	0.4968	0.4886	0.4808	0.4734
2	0.3315	0.3325	0.3325	0.3318	0.3306	0.3290	0.3273	0.3253	0.3232	0.3211
3	0.2260	0.2347	0.2412	0.2460	0.2495	0.2521	0.2540	0.2553	0.2561	0.2565
4	0.1429	0.1586	0.1707	0.1802	0.1878	0.1939	0.1988	0.2027	0.2059	0.2085
5	0.0695	0.0922	0.1099	0.1240	0.1353	0.1447	0.1524	0.1587	0.1641	0.1686
6	0.0000	0.0303	0.0539	0.0727	0.0880	0.1005	0.1109	0.1197	0.1271	0.1334
7	-	-	0.0000	0.0240	0.0433	0.0593	0.0725	0.0837	0.0932	0.1013
8	-	-	-	-	0.0000	0.0196	0.0359	0.0496	0.0612	0.0711
9	-	-	-	-	-	-	0.0000	0.0163	0.0303	0.0422
10	-	-	-	-	-	-	-	-	0.0000	0.0140

i \ n	21	22	23	24	25	26	27	28	29	30
1	0.4643	0.4590	0.4542	0.4493	0.4450	0.4407	0.4366	0.4328	0.4291	0.4254
2	0.3185	0.3156	0.3126	0.3098	0.3069	0.3043	0.3018	0.2992	0.2968	0.2944
3	0.2578	0.2571	0.2563	0.2554	0.2543	0.2533	0.2522	0.2510	0.2499	0.2487
4	0.2119	0.2131	0.2139	0.2145	0.2148	0.2151	0.2152	0.2151	0.2150	0.2148
5	0.1736	0.1764	0.1787	0.1807	0.1822	0.1836	0.1848	0.1857	0.1864	0.1870
6	0.1399	0.1443	0.1480	0.1512	0.1539	0.1563	0.1584	0.1601	0.1616	0.1630
7	0.1092	0.1150	0.1201	0.1245	0.1283	0.1316	0.1346	0.1372	0.1395	0.1415
8	0.0804	0.0878	0.0941	0.0997	0.1046	0.1089	0.1128	0.1162	0.1192	0.1219
9	0.0530	0.0618	0.0696	0.0764	0.0823	0.0876	0.0923	0.0965	0.1002	0.1036
10	0.0263	0.0368	0.0459	0.0539	0.0610	0.0672	0.0728	0.0778	0.0822	0.0862
11	0.0000	0.0122	0.0228	0.0321	0.0403	0.0476	0.0540	0.0598	0.0650	0.0697
12	-	-	0.0000	0.0107	0.0200	0.0284	0.0358	0.0424	0.0483	0.0537
13	-	-	-	-	0.0000	0.0094	0.0178	0.0253	0.0320	0.0381
14	-	-	-	-	-	-	0.0000	0.0084	0.0159	0.0227
15	-	-	-	-	-	-	-	-	0.0000	0.0076

Source: After Shapiro and Wilk, 1965. Used by permission of the Biometrika Trustees.

TABLE A-1. (Continued)

i \ n	31	32	33	34	35	36	37	38	39	40
1	0.4220	0.4188	0.4156	0.4127	0.4096	0.4068	0.4040	0.4015	0.3989	0.3964
2	0.2921	0.2898	0.2876	0.2854	0.2834	0.2813	0.2794	0.2774	0.2755	0.2737
3	0.2475	0.2462	0.2451	0.2439	0.2427	0.2415	0.2403	0.2391	0.2380	0.2368
4	0.2145	0.2141	0.2137	0.2132	0.2127	0.2121	0.2116	0.2110	0.2104	0.2098
5	0.1874	0.1878	0.1880	0.1882	0.1883	0.1883	0.1883	0.1881	0.1880	0.1878
6	0.1641	0.1651	0.1660	0.1667	0.1673	0.1678	0.1683	0.1686	0.1689	0.1691
7	0.1433	0.1449	0.1463	0.1475	0.1487	0.1496	0.1505	0.1513	0.1520	0.1526
8	0.1243	0.1265	0.1284	0.1301	0.1317	0.1331	0.1344	0.1356	0.1366	0.1376
9	0.1066	0.1093	0.1116	0.1140	0.1160	0.1179	0.1196	0.1211	0.1225	0.1237
10	0.0899	0.0931	0.0961	0.0988	0.1013	0.1036	0.1056	0.1075	0.1092	0.1108
11	0.0739	0.0777	0.0812	0.0844	0.0873	0.0900	0.0924	0.0947	0.0967	0.0986
12	0.0585	0.0629	0.0669	0.0706	0.0739	0.0770	0.0798	0.0824	0.0848	0.0870
13	0.0435	0.0485	0.0530	0.0572	0.0610	0.0645	0.0677	0.0706	0.0733	0.0759
14	0.0289	0.0344	0.0395	0.0441	0.0484	0.0523	0.0559	0.0592	0.0622	0.0651
15	0.0144	0.0206	0.0262	0.0314	0.0361	0.0404	0.0444	0.0481	0.0515	0.0546
16	0.0000	0.0068	0.0131	0.0187	0.0239	0.0287	0.0331	0.0372	0.0409	0.0444
17	-	-	0.0000	0.0062	0.0119	0.0172	0.0220	0.0264	0.0305	0.0343
18	-	-	-	-	0.0000	0.0057	0.0110	0.0158	0.0203	0.0244
19	-	-	-	-	-	-	0.0000	0.0053	0.0101	0.0146
20	-	-	-	-	-	-	-	-	0.0000	0.0049

i \ n	41	42	43	44	45	46	47	48	49	50
1	0.3940	0.3917	0.3894	0.3872	0.3850	0.3830	0.3808	0.3789	0.3770	0.3751
2	0.2719	0.2701	0.2684	0.2667	0.2651	0.2635	0.2620	0.2604	0.2589	0.2574
3	0.2357	0.2345	0.2334	0.2323	0.2313	0.2302	0.2291	0.2281	0.2271	0.2260
4	0.2091	0.2085	0.2078	0.2072	0.2065	0.2058	0.2052	0.2045	0.2038	0.2032
5	0.1876	0.1874	0.1871	0.1868	0.1865	0.1862	0.1859	0.1855	0.1851	0.1847
6	0.1693	0.1694	0.1695	0.1695	0.1695	0.1695	0.1695	0.1693	0.1692	0.1691
7	0.1531	0.1535	0.1539	0.1542	0.1545	0.1548	0.1550	0.1551	0.1553	0.1554
8	0.1384	0.1392	0.1398	0.1405	0.1410	0.1415	0.1420	0.1423	0.1427	0.1430
9	0.1249	0.1259	0.1269	0.1278	0.1286	0.1293	0.1300	0.1306	0.1312	0.1317
10	0.1123	0.1136	0.1149	0.1160	0.1170	0.1180	0.1189	0.1197	0.1205	0.1212
11	0.1004	0.1020	0.1035	0.1049	0.1062	0.1073	0.1085	0.1095	0.1105	0.1113
12	0.0891	0.0909	0.0927	0.0943	0.0959	0.0972	0.0986	0.0998	0.1010	0.1020
13	0.0782	0.0804	0.0824	0.0842	0.0860	0.0876	0.0892	0.0906	0.0919	0.0932
14	0.0677	0.0701	0.0724	0.0745	0.0765	0.0783	0.0801	0.0817	0.0832	0.0846
15	0.0575	0.0602	0.0628	0.0651	0.0673	0.0694	0.0713	0.0731	0.0748	0.0764
16	0.0476	0.0506	0.0534	0.0560	0.0584	0.0607	0.0628	0.0648	0.0667	0.0685
17	0.0379	0.0411	0.0442	0.0471	0.0497	0.0522	0.0546	0.0568	0.0588	0.0608
18	0.0283	0.0318	0.0352	0.0383	0.0412	0.0439	0.0465	0.0489	0.0511	0.0532
19	0.0188	0.0227	0.0263	0.0296	0.0328	0.0357	0.0385	0.0411	0.0436	0.0459
20	0.0094	0.0136	0.0175	0.0211	0.0245	0.0277	0.0307	0.0335	0.0361	0.0386
21	0.0000	0.0045	0.0087	0.0126	0.0163	0.0197	0.0229	0.0259	0.0288	0.0314
22	-	-	0.0000	0.0042	0.0081	0.0118	0.0153	0.0185	0.0215	0.0244
23	-	-	-	-	0.0000	0.0039	0.0076	0.0111	0.0143	0.0174
24	-	-	-	-	-	-	0.0000	0.0037	0.0071	0.0104
25	-	-	-	-	-	-	-	-	0.0000	0.0035

TABLE A-2. QUANTILES OF THE SHAPIRO-WILK W TEST FOR NORMALITY

n	$W_{0.01}$	$W_{0.02}$	$W_{0.05}$	$W_{0.10}$	$W_{0.50}$
3	0.753	0.756	0.767	0.789	0.959
4	0.687	0.707	0.748	0.792	0.935
5	0.686	0.715	0.762	0.806	0.927
6	0.713	0.743	0.788	0.826	0.927
7	0.730	0.760	0.803	0.838	0.928
8	0.749	0.778	0.818	0.851	0.932
9	0.764	0.791	0.829	0.859	0.935
10	0.781	0.806	0.842	0.869	0.938
11	0.792	0.817	0.850	0.876	0.940
12	0.805	0.828	0.859	0.883	0.943
13	0.814	0.837	0.866	0.889	0.945
14	0.825	0.846	0.874	0.895	0.947
15	0.835	0.855	0.881	0.901	0.950
16	0.844	0.863	0.887	0.906	0.952
17	0.851	0.869	0.892	0.910	0.954
18	0.858	0.874	0.897	0.914	0.956
19	0.863	0.879	0.901	0.917	0.957
20	0.868	0.884	0.905	0.920	0.959
21	0.873	0.888	0.908	0.923	0.960
22	0.878	0.892	0.911	0.926	0.961
23	0.881	0.895	0.914	0.928	0.962
24	0.884	0.898	0.916	0.930	0.963
25	0.886	0.901	0.918	0.931	0.964
26	0.891	0.904	0.920	0.933	0.965
27	0.894	0.906	0.923	0.935	0.965
28	0.896	0.908	0.924	0.936	0.966
29	0.898	0.910	0.926	0.937	0.966
30	0.900	0.912	0.927	0.939	0.967
31	0.902	0.914	0.929	0.940	0.967
32	0.904	0.915	0.930	0.941	0.968
33	0.906	0.917	0.931	0.942	0.968
34	0.908	0.919	0.933	0.943	0.969
35	0.910	0.920	0.934	0.944	0.969
36	0.912	0.922	0.935	0.945	0.970
37	0.914	0.924	0.936	0.946	0.970
38	0.916	0.925	0.938	0.947	0.971
39	0.917	0.927	0.939	0.948	0.971
40	0.919	0.928	0.940	0.949	0.972
41	0.920	0.929	0.941	0.950	0.972
42	0.922	0.930	0.942	0.951	0.972
43	0.923	0.932	0.943	0.951	0.973
44	0.924	0.933	0.944	0.952	0.973
45	0.926	0.934	0.945	0.953	0.973
46	0.927	0.935	0.945	0.953	0.974
47	0.928	0.936	0.946	0.954	0.974
48	0.929	0.937	0.947	0.954	0.974
49	0.929	0.937	0.947	0.955	0.974
50	0.930	0.938	0.947	0.955	0.974

Values of W such that 100 p percent of the distribution of W is less than Wp.

Source: After Shapiro and Wilk, 1965. Used by permission of the Biometrika Trustees.

TABLE A-3. TABLE OF k USED IN CALCULATING TOLERANCE INTERVALS FOR A NORMALLY DISTRIBUTED VARIABLE (α = 0.05, P_0, and sample size n)

n	P_0 0.25	0.1	0.05	0.010
2	11.763	20.581	26.260	37.094
3	3.806	6.155	7.656	10.553
4	2.618	4.162	5.144	7.042
5	2.150	3.407	4.203	5.741
6	1.895	3.006	3.708	5.062
7	1.732	2.755	3.399	4.642
8	1.618	2.582	3.187	4.354
9	1.532	2.454	3.031	4.143
10	1.465	2.355	2.911	3.981
11	1.411	2.275	2.815	3.852
12	1.366	2.210	2.736	3.747
13	1.328	2.155	2.671	3.659
14	1.296	2.109	2.614	3.585
15	1.268	2.068	2.566	3.520
16	1.243	2.033	2.524	3.464
17	1.220	2.002	2.486	3.414
18	1.201	1.974	2.453	3.370
19	1.183	1.949	2.423	3.331
20	1.166	1.926	2.396	3.295
21	1.152	1.905	2.371	3.263
22	1.138	1.886	2.349	3.233
23	1.125	1.869	2.328	3.206
24	1.114	1.853	2.309	3.181
25	1.103	1.838	2.292	3.158
26	1.093	1.824	2.275	3.136
27	1.083	1.811	2.260	3.116
28	1.075	1.799	2.246	3.098
29	1.066	1.788	2.232	3.080
30	1.058	1.777	2.220	3.064
35	1.025	1.732	2.167	2.995
40	0.999	1.697	2.125	2.941
50	0.960	1.646	2.065	2.862
70	0.911	1.581	1.990	2.765
100	0.870	1.527	1.927	2.684
200	0.809	1.450	1.837	2.570
500	0.758	1.385	1.763	2.475
infinity	0.674	1.282	1.645	2.326

From U.S. EPA (1988).

TABLE A-4. TABLE OF ONE-SIDED CONFIDENCE LIMIT VALUES (for selected α and degrees of freedom)

Use alpha to determine which column to use. Use the degrees of freedom to determine which row to use. The t value will be found at the intersection of the row and column. For values of degrees of freedom not in the table, interpolate between those values provided.

	α for determining $t_{1-\alpha,df}$							
Degrees of Freedom df	.25	.10	.05	.025	.01	.005	.0025	.001
1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160
400	0.675	1.284	1.649	1.966	2.336	2.588	2.823	3.111
infinite	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090

TABLE A-5. BINOMIAL DISTRIBUTION

n	y	p = .05	.10	.15	.20	.25	.30	.35	.40	.45
1	0	.9500	.9000	.8500	.8000	.7500	.7000	.6500	.6000	.5500
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0	.9025	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025
	1	.9975	.9900	.9775	.9600	.9375	.9100	.8775	.8400	.7975
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	.8574	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664
	1	.9928	.9720	.9392	.8960	.8438	.7840	.7182	.6480	.5748
	2	.9999	.9990	.9966	.9920	.9844	.9730	.9571	.9360	.9089
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915
	1	.9860	.9477	.8905	.8192	.7383	.6517	.5630	.4752	.3910
	2	.9995	.9963	.9880	.9728	.9492	.9163	.8735	.8208	.7585
	3	1.0000	.9999	.9995	.9984	.9961	.9919	.9850	.9744	.9590
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0	.7738	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503
	1	.9774	.9185	.8352	.7373	.6328	.5282	.4284	.3370	.2562
	2	.9988	.9914	.9734	.9421	.8965	.8369	.7648	.6826	.5931
	3	1.0000	.9995	.9978	.9933	.9844	.9692	.9460	.9130	.8688
	4	1.0000	1.0000	.9999	.9997	.9990	.9976	.9947	.9898	.9815
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0	.7351	.5314	.3771	.2621	.1780	.1176	.0754	.0467	.0277
	1	.9672	.8857	.7765	.6554	.5339	.4202	.3191	.2333	.1636
	2	.9978	.9842	.9527	.9011	.8306	.7443	.6471	.5443	.4415
	3	.9999	.9987	.9941	.9830	.9624	.9295	.8826	.8208	.7447
	4	1.0000	.9999	.9996	.9984	.9954	.9891	.9777	.9590	.9308
	5	1.0000	1.0000	1.0000	.9999	.9998	.9993	.9982	.9959	.9917
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0	.6983	.4783	.3206	.2097	.1335	.0824	.0490	.0280	.0152
	1	.9556	.8503	.7166	.5767	.4449	.3294	.2338	.1586	.1024
	2	.9962	.9743	.9262	.8520	.7564	.6471	.5323	.4199	.3164
	3	.9998	.9973	.9879	.9667	.9294	.8740	.8002	.7102	.6083
	4	1.0000	.9998	.9988	.9953	.9871	.9712	.9444	.9037	.8471
	5	1.0000	1.0000	.9999	.9996	.9987	.9962	.9910	.9812	.9643
	6	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9994	.9984	.9963
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	y	p = .50	.55	.60	.65	.70	.75	.80	.85	.90	.95
1	0	.5000	.4500	.4000	.3500	.3000	.2500	.2000	.1500	.1000	.0500
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0	.2500	.2025	.1600	.1225	.0900	.0625	.0400	.0225	.0100	.0025
	1	.7500	.6975	.6400	.5775	.5100	.4375	.3600	.2775	.1900	.0975
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	.1250	.0911	.0640	.0429	.0270	.0156	.0080	.0034	.0010	.0001
	1	.5000	.4252	.3520	.2818	.2160	.1562	.1040	.0608	.0280	.0072
	2	.8750	.8336	.7840	.7254	.6570	.5781	.4880	.3859	.2710	.1426
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0	.0625	.0410	.0256	.0150	.0081	.0039	.0016	.0005	.0001	.0000
	1	.3125	.2415	.1792	.1265	.0837	.0508	.0272	.0120	.0037	.0005
	2	.6875	.6090	.5248	.4370	.3483	.2617	.1808	.1095	.0523	.0140
	3	.9375	.9085	.8704	.8215	.7599	.6836	.5904	.4780	.3439	.1855
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0	.0312	.0185	.0102	.0053	.0024	.0010	.0003	.0001	.0000	.0000
	1	.1875	.1312	.0870	.0540	.0308	.0156	.0067	.0022	.0005	.0000
	2	.5000	.4069	.3174	.2352	.1631	.1035	.0579	.0266	.0086	.0012
	3	.8125	.7438	.6630	.5716	.4718	.3672	.2627	.1648	.0815	.0226
	4	.9688	.9497	.9222	.8840	.8319	.7627	.6723	.5563	.4095	.2262
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0	.0156	.0083	.0041	.0018	.0007	.0002	.0001	.0000	.0000	.0000
	1	.1094	.0692	.0410	.0223	.0109	.0046	.0016	.0004	.0001	.0000
	2	.3438	.2553	.1792	.1174	.0705	.0376	.0170	.0059	.0013	.0001
	3	.6562	.5585	.4557	.3529	.2557	.1694	.0989	.0473	.0158	.0022
	4	.8906	.8364	.7667	.6809	.5798	.4661	.3446	.2235	.1143	.0328
	5	.9844	.9723	.9533	.9246	.8824	.8220	.7379	.6229	.4686	.2649
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0	.0078	.0037	.0016	.0006	.0002	.0001	.0000	.0000	.0000	.0000
	1	.0625	.0357	.0188	.0090	.0038	.0013	.0004	.0001	.0000	.0000
	2	.2266	.1529	.0963	.0556	.0288	.0129	.0047	.0012	.0002	.0000
	3	.5000	.3917	.2898	.1998	.1260	.0706	.0333	.0121	.0027	.0002
	4	.7734	.6836	.5801	.4677	.3529	.2436	.1480	.0738	.0257	.0038
	5	.9375	.8976	.8414	.7662	.6706	.5551	.4233	.2834	.1497	.0444
	6	.9922	.9848	.9720	.9510	.9176	.8665	.7903	.6794	.5217	.3017
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	y	p = .05	.10	.15	.20	.25	.30	.35	.40	.45
8	0	.6634	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084
	1	.9428	.8131	.6572	.5033	.3671	.2553	.1691	.1064	.0632
	2	.9942	.9619	.8948	.7969	.6785	.5518	.4278	.3154	.2201
	3	.9996	.9950	.9786	.9437	.8862	.8059	.7064	.5941	.4770
	4	1.0000	.9996	.9971	.9896	.9727	.9420	.8939	.8263	.7396
	5	1.0000	1.0000	.9998	.9988	.9958	.9887	.9747	.9502	.9115
	6	1.0000	1.0000	1.0000	.9999	.9996	.9987	.9964	.9915	.9819
	7	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9993	.9983
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0	.6302	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046
	1	.9288	.7748	.5995	.4362	.3003	.1960	.1211	.0705	.0385
	2	.9916	.9470	.8591	.7382	.6007	.4628	.3373	.2318	.1495
	3	.9994	.9917	.9661	.9144	.8343	.7297	.6089	.4826	.3614
	4	1.0000	.9991	.9944	.9804	.9511	.9012	.8283	.7334	.6214
	5	1.0000	.9999	.9994	.9969	.9900	.9747	.9464	.9006	.8342
	6	1.0000	1.0000	1.0000	.9997	.9987	.9957	.9888	.9750	.9502
	7	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9986	.9962	.9909
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9992
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0	.5987	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025
	1	.9139	.7361	.5443	.3758	.2440	.1493	.0860	.0464	.0233
	2	.9885	.9298	.8202	.6778	.5256	.3828	.2616	.1673	.0996
	3	.9990	.9872	.9500	.8791	.7759	.6496	.5138	.3823	.2660
	4	.9999	.9984	.9901	.9672	.9219	.8497	.7515	.6331	.5044
	5	1.0000	.9999	.9986	.9936	.9803	.9527	.9051	.8338	.7384
	6	1.0000	1.0000	.9999	.9991	.9965	.9894	.9740	.9452	.8980
	7	1.0000	1.0000	1.0000	.9999	.9996	.9984	.9952	.9877	.9726
	8	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9983	.9955
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0	.5688	.3138	.1673	.0859	.0422	.0198	.0088	.0036	.0014
	1	.8981	.6974	.4922	.3221	.1971	.1130	.0606	.0302	.0139
	2	.9848	.9104	.7788	.6174	.4552	.3127	.2001	.1189	.0652
	3	.9984	.9815	.9306	.8389	.7133	.5696	.4256	.2963	.1911
	4	.9999	.9972	.9841	.9496	.8854	.7897	.6683	.5328	.3971
	5	1.0000	.9997	.9973	.9883	.9657	.9218	.8513	.7535	.6331
	6	1.0000	1.0000	.9997	.9980	.9924	.9784	.9499	.9006	.8262
	7	1.0000	1.0000	1.0000	.9998	.9988	.9957	.9878	.9707	.9390
	8	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9980	.9941	.9852
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9993	.9978
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	y	p = .50	.55	.60	.65	.70	.75	.80	.85	.90	.95
8	0	.0039	.0017	.0007	.0002	.0001	.0000	.0000	.0000	.0000	.0000
	1	.0352	.0181	.0085	.0036	.0013	.0004	.0001	.0000	.0000	.0000
	2	.1445	.0885	.0498	.0253	.0113	.0042	.0012	.0002	.0000	.0000
	3	.3633	.2604	.1737	.1061	.0580	.0273	.0104	.0029	.0004	.0000
	4	.6367	.5230	.4059	.2936	.1941	.1138	.0563	.0214	.0050	.0004
	5	.8555	.7799	.6846	.5722	.4482	.3215	.2031	.1052	.0381	.0058
	6	.9648	.9368	.8936	.8309	.7447	.6329	.4967	.3428	.1869	.0572
	7	.9961	.9916	.9832	.9681	.9424	.8999	.8322	.7275	.5695	.3366
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0	.0020	.0008	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0195	.0091	.0038	.0014	.0004	.0001	.0000	.0000	.0000	.0000
	2	.0898	.0498	.0250	.0112	.0043	.0013	.0003	.0000	.0000	.0000
	3	.2539	.1658	.0994	.0536	.0253	.0100	.0031	.0006	.0001	.0000
	4	.5000	.3786	.2666	.1717	.0988	.0489	.0196	.0056	.0009	.0000
	5	.7461	.6386	.5174	.3911	.2703	.1657	.0856	.0339	.0083	.0006
	6	.9102	.8505	.7682	.6627	.5372	.3993	.2618	.1409	.0530	.0084
	7	.9805	.9615	.9295	.8789	.8040	.6997	.5638	.4005	.2252	.0712
	8	.9980	.9954	.9899	.9793	.9596	.9249	.8658	.7684	.6126	.3698
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0	.0010	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0107	.0045	.0017	.0005	.0001	.0000	.0000	.0000	.0000	.0000
	2	.0547	.0274	.0123	.0048	.0016	.0004	.0001	.0000	.0000	.0000
	3	.1719	.1020	.0548	.0260	.0106	.0035	.0009	.0001	.0000	.0000
	4	.3770	.2616	.1662	.0949	.0473	.0197	.0064	.0014	.0001	.0000
	5	.6230	.4956	.3669	.2485	.1503	.0781	.0328	.0099	.0016	.0001
	6	.8281	.7340	.6177	.4862	.3504	.2241	.1209	.0500	.0128	.0010
	7	.9453	.9004	.8327	.7384	.6172	.4744	.3222	.1798	.0702	.0115
	8	.9893	.9767	.9536	.9140	.8507	.7560	.6242	.4557	.2639	.0861
	9	.9990	.9975	.9940	.9865	.9718	.9437	.8926	.8031	.6513	.4013
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0	.0005	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0059	.0022	.0007	.0002	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0327	.0148	.0059	.0020	.0006	.0001	.0000	.0000	.0000	.0000
	3	.1133	.0610	.0293	.0122	.0043	.0012	.0002	.0000	.0000	.0000
	4	.2744	.1738	.0994	.0501	.0216	.0076	.0020	.0003	.0000	.0000
	5	.5000	.3669	.2465	.1487	.0782	.0343	.0117	.0027	.0003	.0000
	6	.7256	.6029	.4672	.3317	.2103	.1146	.0504	.0159	.0028	.0001
	7	.8867	.8089	.7037	.5744	.4304	.2867	.1611	.0694	.0185	.0016
	8	.9673	.9348	.8811	.7999	.6873	.5448	.3826	.2212	.0896	.0152
	9	.9941	.9861	.9698	.9394	.8870	.8029	.6779	.5078	.3026	.1019
	10	.9995	.9986	.9964	.9912	.9802	.9578	.9141	.8327	.6862	.4312
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	y	p = .05	.10	.15	.20	.25	.30	.35	.40	.45
12	0	.5404	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008
	1	.8816	.6590	.4435	.2749	.1584	.0850	.0424	.0196	.0083
	2	.9804	.8891	.7358	.5583	.3907	.2528	.1513	.0834	.0421
	3	.9978	.9744	.9078	.7946	.6488	.4925	.3467	.2253	.1345
	4	.9998	.9957	.9761	.9274	.8424	.7237	.5833	.4382	.3044
	5	1.0000	.9995	.9954	.9806	.9456	.8822	.7873	.6652	.5269
	6	1.0000	.9999	.9993	.9961	.9857	.9614	.9154	.8418	.7393
	7	1.0000	1.0000	.9999	.9994	.9972	.9905	.9745	.9427	.8883
	8	1.0000	1.0000	1.0000	.9999	.9996	.9983	.9944	.9847	.9644
	9	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9992	.9972	.9921
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9989
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	0	.5133	.2542	.1209	.0550	.0238	.0097	.0037	.0013	.0004
	1	.8646	.6213	.3983	.2336	.1267	.0637	.0296	.0126	.0049
	2	.9755	.8661	.6920	.5017	.3326	.2025	.1132	.0579	.0269
	3	.9969	.9658	.8820	.7473	.5843	.4206	.2783	.1686	.0929
	4	.9997	.9935	.9658	.9009	.7940	.6543	.5005	.3530	.2279
	5	1.0000	.9991	.9925	.9700	.9198	.8346	.7159	.5744	.4268
	6	1.0000	.9999	.9987	.9930	.9757	.9376	.8705	.7712	.6437
	7	1.0000	1.0000	.9998	.9988	.9944	.9818	.9538	.9023	.8212
	8	1.0000	1.0000	1.0000	.9998	.9990	.9960	.9874	.9679	.9302
	9	1.0000	1.0000	1.0000	1.0000	.9999	.9993	.9975	.9922	.9797
	10	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9987	.9959
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	0	.4877	.2288	.1028	.0440	.0178	.0068	.0024	.0008	.0002
	1	.8470	.5846	.3567	.1979	.1010	.0475	.0205	.0081	.0029
	2	.9699	.8416	.6479	.4481	.2811	.1608	.0839	.0398	.0170
	3	.9958	.9559	.8535	.6982	.5213	.3552	.2205	.1243	.0632
	4	.9996	.9908	.9533	.8702	.7415	.5842	.4227	.2793	.1672
	5	1.0000	.9985	.9885	.9561	.8883	.7805	.6405	.4859	.3373
	6	1.0000	.9998	.9978	.9884	.9617	.9067	.8164	.6925	.5461
	7	1.0000	1.0000	.9997	.9976	.9897	.9685	.9247	.8499	.7414
	8	1.0000	1.0000	1.0000	.9996	.9978	.9917	.9757	.9417	.8811
	9	1.0000	1.0000	1.0000	1.0000	.9997	.9983	.9940	.9825	.9574
	10	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9989	.9961	.9886
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9978
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	y	p = .50	.55	.60	.65	.70	.75	.80	.85	.90	.95
12	0	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0032	.0011	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0193	.0079	.0028	.0008	.0002	.0000	.0000	.0000	.0000	.0000
	3	.0730	.0356	.0153	.0056	.0017	.0004	.0001	.0000	.0000	.0000
	4	.1938	.1117	.0573	.0255	.0095	.0028	.0006	.0001	.0000	.0000
	5	.3872	.2607	.1582	.0846	.0386	.0143	.0039	.0007	.0001	.0000
	6	.6128	.4731	.3348	.2127	.1178	.0544	.0194	.0046	.0005	.0000
	7	.8062	.6956	.5618	.4167	.2763	.1576	.0726	.0239	.0043	.0002
	8	.9270	.8655	.7747	.6533	.5075	.3512	.2054	.0922	.0256	.0022
	9	.9807	.9579	.9166	.8487	.7472	.6093	.4417	.2642	.1109	.0196
	10	.9968	.9917	.9804	.9576	.9150	.8416	.7251	.5565	.3410	.1184
	11	.9998	.9992	.9978	.9943	.9862	.9683	.9313	.8578	.7176	.4596
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0017	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0112	.0041	.0013	.0003	.0001	.0000	.0000	.0000	.0000	.0000
	3	.0461	.0203	.0078	.0025	.0007	.0001	.0000	.0000	.0000	.0000
	4	.1334	.0698	.0321	.0126	.0040	.0010	.0002	.0000	.0000	.0000
	5	.2905	.1788	.0977	.0462	.0182	.0056	.0012	.0002	.0000	.0000
	6	.5000	.3563	.2288	.1295	.0624	.0243	.0070	.0013	.0001	.0000
	7	.7095	.5732	.4256	.2841	.1654	.0802	.0300	.0075	.0009	.0000
	8	.8666	.7721	.6470	.4995	.3457	.2060	.0991	.0342	.0065	.0003
	9	.9539	.9071	.8314	.7217	.5794	.4157	.2527	.1180	.0342	.0031
	10	.9888	.9731	.9421	.8868	.7975	.6674	.4983	.3080	.1339	.0245
	11	.9983	.9951	.9874	.9704	.9363	.8733	.7664	.6017	.3787	.1354
	12	.9999	.9996	.9987	.9963	.9903	.9762	.9450	.8791	.7458	.4867
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0009	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0065	.0022	.0006	.0001	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0287	.0114	.0039	.0011	.0002	.0000	.0000	.0000	.0000	.0000
	4	.0898	.0426	.0175	.0060	.0017	.0003	.0000	.0000	.0000	.0000
	5	.2120	.1189	.0583	.0243	.0083	.0022	.0004	.0000	.0000	.0000
	6	.3953	.2586	.1501	.0753	.0315	.0103	.0024	.0003	.0000	.0000
	7	.6047	.4539	.3075	.1836	.0933	.0383	.0116	.0022	.0002	.0000
	8	.7880	.6627	.5141	.3595	.2195	.1117	.0439	.0115	.0015	.0000
	9	.9102	.8328	.7207	.5773	.4158	.2585	.1298	.0467	.0092	.0004
	10	.9713	.9368	.8757	.7795	.6448	.4787	.3018	.1465	.0441	.0042
	11	.9935	.9830	.9602	.9161	.8392	.7189	.5519	.3521	.1584	.0301
	12	.9991	.9971	.9919	.9795	.9525	.8990	.8021	.6433	.4154	.1530
	13	.9999	.9998	.9992	.9976	.9932	.9822	.9560	.8972	.7712	.5123
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	y	p = .05	.10	.15	.20	.25	.30	.35	.40	.45
15	0	.4633	.2059	.0874	.0352	.0134	.0047	.0016	.0005	.0001
	1	.8290	.5490	.3186	.1671	.0802	.0353	.0142	.0052	.0017
	2	.9638	.8159	.6042	.3980	.2361	.1268	.0617	.0271	.0107
	3	.9945	.9444	.8227	.6482	.4613	.2969	.1727	.0905	.0424
	4	.9994	.9873	.9383	.8358	.6865	.5155	.3519	.2173	.1204
	5	.9999	.9978	.9832	.9389	.8516	.7216	.5643	.4032	.2608
	6	1.0000	.9997	.9964	.9819	.9434	.8689	.7548	.6098	.4522
	7	1.0000	1.0000	.9994	.9958	.9827	.9500	.8868	.7869	.6535
	8	1.0000	1.0000	.9999	.9992	.9958	.9848	.9578	.9050	.8182
	9	1.0000	1.0000	1.0000	.9999	.9992	.9963	.9876	.9662	.9231
	10	1.0000	1.0000	1.0000	1.0000	.9999	.9993	.9972	.9907	.9745
	11	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9981	.9937
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9989
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	0	.4401	.1853	.0743	.0281	.0100	.0033	.0010	.0003	.0001
	1	.8108	.5147	.2839	.1407	.0635	.0261	.0098	.0033	.0010
	2	.9571	.7892	.5614	.3518	.1971	.0994	.0451	.0183	.0066
	3	.9930	.9316	.7899	.5981	.4050	.2459	.1339	.0651	.0281
	4	.9991	.9830	.9209	.7982	.6302	.4499	.2892	.1666	.0853
	5	.9999	.9967	.9765	.9183	.8103	.6598	.4900	.3288	.1976
	6	1.0000	.9995	.9944	.9733	.9204	.8247	.6881	.5272	.3660
	7	1.0000	.9999	.9989	.9930	.9729	.9256	.8406	.7161	.5629
	8	1.0000	1.0000	.9998	.9985	.9925	.9743	.9329	.8577	.7441
	9	1.0000	1.0000	1.0000	.9998	.9984	.9929	.9771	.9417	.8759
	10	1.0000	1.0000	1.0000	1.0000	.9997	.9984	.9938	.9809	.9514
	11	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9987	.9951	.9851
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9991	.9965
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	*y*	*p* = .50	.55	.60	.65	.70	.75	.80	.85	.90	.95
15	0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0037	.0011	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0176	.0063	.0019	.0005	.0001	.0000	.0000	.0000	.0000	.0000
	4	.0592	.0255	.0093	.0028	.0007	.0001	.0000	.0000	.0000	.0000
	5	.1509	.0769	.0338	.0124	.0037	.0008	.0001	.0000	.0000	.0000
	6	.3036	.1818	.0950	.0422	.0152	.0042	.0008	.0001	.0000	.0000
	7	.5000	.3465	.2131	.1132	.0500	.0173	.0042	.0006	.0000	.0000
	8	.6964	.5478	.3902	.2452	.1311	.0566	.0181	.0036	.0003	.0000
	9	.8491	.7392	.5968	.4357	.2784	.1484	.0611	.0168	.0022	.0001
	10	.9408	.8796	.7827	.6481	.4845	.3135	.1642	.0617	.0127	.0006
	11	.9824	.9576	.9095	.8273	.7031	.5387	.3518	.1773	.0556	.0055
	12	.9963	.9893	.9729	.9383	.8732	.7639	.6020	.3958	.1841	.0362
	13	.9995	.9983	.9948	.9858	.9647	.9198	.8329	.6814	.4510	.1710
	14	1.0000	.9999	.9995	.9984	.9953	.9866	.9648	.9126	.7941	.5367
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0021	.0006	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0106	.0035	.0009	.0002	.0000	.0000	.0000	.0000	.0000	.0000
	4	.0384	.0149	.0049	.0013	.0003	.0000	.0000	.0000	.0000	.0000
	5	.1051	.0486	.0191	.0062	.0016	.0003	.0000	.0000	.0000	.0000
	6	.2272	.1241	.0583	.0229	.0071	.0016	.0002	.0000	.0000	.0000
	7	.4018	.2559	.1423	.0671	.0257	.0075	.0015	.0002	.0000	.0000
	8	.5982	.4371	.2839	.1594	.0744	.0271	.0070	.0011	.0001	.0000
	9	.7728	.6340	.4728	.3119	.1753	.0796	.0267	.0056	.0005	.0000
	10	.8949	.8024	.6712	.5100	.3402	.1897	.0817	.0235	.0033	.0001
	11	.9616	.9147	.8334	.7108	.5501	.3698	.2018	.0791	.0170	.0009
	12	.9894	.9719	.9349	.8661	.7541	.5950	.4019	.2101	.0684	.0070
	13	.9979	.9934	.9817	.9549	.9006	.8029	.6482	.4386	.2108	.0429
	14	.9997	.9990	.9967	.9902	.9739	.9365	.8593	.7161	.4853	.1892
	15	1.0000	.9999	.9997	.9990	.9967	.9900	.9719	.9257	.8147	.5599
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	*y*	*p* = .05	.10	.15	.20	.25	.30	.35	.40	.45
17	0	.4181	.1668	.0631	.0225	.0075	.0023	.0007	.0002	.0000
	1	.7922	.4818	.2525	.1182	.0501	.0193	.0067	.0021	.0006
	2	.9497	.7618	.5198	.3096	.1637	.0774	.0327	.0123	.0041
	3	.9912	.9174	.7556	.5489	.3530	.2019	.1028	.0464	.0184
	4	.9988	.9779	.9013	.7582	.5739	.3887	.2348	.1260	.0596
	5	.9999	.9953	.9681	.8943	.7653	.5968	.4197	.2639	.1471
	6	1.0000	.9992	.9917	.9623	.8929	.7752	.6188	.4478	.2902
	7	1.0000	.9999	.9983	.9891	.9598	.8954	.7872	.6405	.4743
	8	1.0000	1.0000	.9997	.9974	.9876	.9597	.9006	.8011	.6626
	9	1.0000	1.0000	1.0000	.9995	.9969	.9873	.9617	.9081	.8166
	10	1.0000	1.0000	1.0000	.9999	.9994	.9968	.9880	.9652	.9174
	11	1.0000	1.0000	1.0000	1.0000	.9999	.9993	.9970	.9894	.9699
	12	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9975	.9914
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9981
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	0	.3972	.1501	.0536	.0180	.0056	.0016	.0004	.0001	.0000
	1	.7735	.4503	.2241	.0991	.0395	.0142	.0046	.0013	.0003
	2	.9419	.7338	.4797	.2713	.1353	.0600	.0236	.0082	.0025
	3	.9891	.9018	.7202	.5010	.3057	.1646	.0783	.0328	.0120
	4	.9985	.9718	.8794	.7164	.5187	.3327	.1886	.0942	.0411
	5	.9998	.9936	.9581	.8671	.7175	.5344	.3550	.2088	.1077
	6	1.0000	.9988	.9882	.9487	.8610	.7217	.5491	.3743	.2258
	7	1.0000	.9998	.9973	.9837	.9431	.8593	.7283	.5634	.3915
	8	1.0000	1.0000	.9995	.9957	.9807	.9404	.8609	.7368	.5778
	9	1.0000	1.0000	.9999	.9991	.9946	.9790	.9403	.8653	.7473
	10	1.0000	1.0000	1.0000	.9998	.9988	.9939	.9788	.9424	.8720
	11	1.0000	1.0000	1.0000	1.0000	.9998	.9986	.9938	.9797	.9463
	12	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9986	.9942	.9817
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9987	.9951
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9990
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	y	p = .50	.55	.60	.65	.70	.75	.80	.85	.90	.95
17	0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0012	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0064	.0019	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000
	4	.0245	.0086	.0025	.0006	.0001	.0000	.0000	.0000	.0000	.0000
	5	.0717	.0301	.0106	.0030	.0007	.0001	.0000	.0000	.0000	.0000
	6	.1662	.0826	.0348	.0120	.0032	.0006	.0001	.0000	.0000	.0000
	7	.3145	.1834	.0919	.0383	.0127	.0031	.0005	.0000	.0000	.0000
	8	.5000	.3374	.1989	.0994	.0403	.0124	.0026	.0003	.0000	.0000
	9	.6855	.5257	.3595	.2128	.1046	.0402	.0109	.0017	.0001	.0000
	10	.8338	.7098	.5522	.3812	.2248	.1071	.0377	.0083	.0008	.0000
	11	.9283	.8529	.7361	.5803	.4032	.2347	.1057	.0319	.0047	.0001
	12	.9755	.9404	.8740	.7652	.6113	.4261	.2418	.0987	.0221	.0012
	13	.9936	.9816	.9536	.8972	.7981	.6470	.4511	.2444	.0826	.0088
	14	.9988	.9959	.9877	.9673	.9226	.8363	.6904	.4802	.2382	.0503
	15	.9999	.9994	.9979	.9933	.9807	.9499	.8818	.7475	.5182	.2078
	16	1.0000	1.0000	.9998	.9993	.9977	.9925	.9775	.9369	.8332	.5819
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0007	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0038	.0010	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	4	.0154	.0049	.0013	.0003	.0000	.0000	.0000	.0000	.0000	.0000
	5	.0481	.0183	.0058	.0014	.0003	.0000	.0000	.0000	.0000	.0000
	6	.1189	.0537	.0203	.0062	.0014	.0002	.0000	.0000	.0000	.0000
	7	.2403	.1280	.0576	.0212	.0061	.0012	.0002	.0000	.0000	.0000
	8	.4073	.2527	.1347	.0597	.0210	.0054	.0009	.0001	.0000	.0000
	9	.5927	.4222	.2632	.1391	.0596	.0193	.0043	.0005	.0000	.0000
	10	.7597	.6085	.4366	.2717	.1407	.0569	.0163	.0027	.0002	.0000
	11	.8811	.7742	.6257	.4509	.2783	.1390	.0513	.0118	.0012	.0000
	12	.9519	.8923	.7912	.6450	.4656	.2825	.1329	.0419	.0064	.0002
	13	.9846	.9589	.9058	.8114	.6673	.4813	.2836	.1206	.0282	.0015
	14	.9962	.9880	.9672	.9217	.8354	.6943	.4990	.2798	.0982	.0109
	15	.9993	.9975	.9918	.9764	.9400	.8647	.7287	.5203	.2662	.0581
	16	.9999	.9997	.9987	.9954	.9858	.9605	.9009	.7759	.5497	.2265
	17	1.0000	1.0000	.9999	.9996	.9984	.9944	.9820	.9464	.8499	.6028
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	*y*	*p* = .05	.10	.15	.20	.25	.30	.35	.40	.45
19	0	.3774	.1351	.0456	.0144	.0042	.0011	.0003	.0001	.0000
	1	.7547	.4203	.1985	.0829	.0310	.0104	.0031	.0008	.0002
	2	.9335	.7054	.4413	.2369	.1113	.0462	.0170	.0055	.0015
	3	.9869	.8850	.6841	.4551	.2631	.1332	.0591	.0230	.0077
	4	.9980	.9648	.8556	.6733	.4654	.2822	.1500	.0696	.0280
	5	.9998	.9914	.9463	.8369	.6678	.4739	.2968	.1629	.0777
	6	1.0000	.9983	.9837	.9324	.8251	.6655	.4812	.3081	.1727
	7	1.0000	.9997	.9959	.9767	.9225	.8180	.6656	.4878	.3169
	8	1.0000	1.0000	.9992	.9933	.9713	.9161	.8145	.6675	.4940
	9	1.0000	1.0000	.9999	.9984	.9911	.9674	.9125	.8139	.6710
	10	1.0000	1.0000	1.0000	.9997	.9977	.9895	.9653	.9115	.8159
	11	1.0000	1.0000	1.0000	1.0000	.9995	.9972	.9886	.9648	.9129
	12	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9969	.9884	.9658
	13	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9993	.9969	.9891
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9972
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	.3585	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000
	1	.7358	.3917	.1756	.0692	.0243	.0076	.0021	.0005	.0001
	2	.9245	.6769	.4049	.2061	.0913	.0355	.0121	.0036	.0009
	3	.9841	.8670	.6477	.4114	.2252	.1071	.0444	.0160	.0049
	4	.9974	.9568	.8298	.6296	.4148	.2375	.1182	.0510	.0189
	5	.9997	.9887	.9327	.8042	.6172	.4164	.2454	.1256	.0553
	6	1.0000	.9976	.9781	.9133	.7858	.6080	.4166	.2500	.1299
	7	1.0000	.9996	.9941	.9679	.8982	.7723	.6010	.4159	.2520
	8	1.0000	.9999	.9987	.9900	.9591	.8867	.7624	.5956	.4143
	9	1.0000	1.0000	.9998	.9974	.9861	.9520	.8782	.7553	.5914
	10	1.0000	1.0000	1.0000	.9994	.9961	.9829	.9468	.8725	.7507
	11	1.0000	1.0000	1.0000	.9999	.9991	.9949	.9804	.9435	.8692
	12	1.0000	1.0000	1.0000	1.0000	.9998	.9987	.9940	.9790	.9420
	13	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9985	.9935	.9786
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9984	.9936
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9985
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9997
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-5. (Continued)

n	y	p = .50	.55	.60	.65	.70	.75	.80	.85	.90	.95
19	0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0004	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0022	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	4	.0096	.0028	.0006	.0001	.0000	.0000	.0000	.0000	.0000	.0000
	5	.0318	.0109	.0031	.0007	.0001	.0000	.0000	.0000	.0000	.0000
	6	.0835	.0342	.0116	.0031	.0006	.0001	.0000	.0000	.0000	.0000
	7	.1796	.0871	.0352	.0114	.0028	.0005	.0000	.0000	.0000	.0000
	8	.3238	.1841	.0885	.0347	.0105	.0023	.0003	.0000	.0000	.0000
	9	.5000	.3290	.1861	.0875	.0326	.0089	.0016	.0001	.0000	.0000
	10	.6762	.5060	.3325	.1855	.0839	.0287	.0067	.0008	.0000	.0000
	11	.8204	.6831	.5122	.3344	.1820	.0775	.0233	.0041	.0003	.0000
	12	.9165	.8273	.6919	.5188	.3345	.1749	.0676	.0163	.0017	.0000
	13	.9682	.9223	.8371	.7032	.5261	.3322	.1631	.0537	.0086	.0002
	14	.9904	.9720	.9304	.8500	.7178	.5346	.3267	.1444	.0352	.0020
	15	.9978	.9923	.9770	.9409	.8668	.7369	.5449	.3159	.1150	.0132
	16	.9996	.9985	.9945	.9830	.9538	.8887	.7631	.5587	.2946	.0665
	17	1.0000	.9998	.9992	.9969	.9896	.9690	.9171	.8015	.5797	.2453
	18	1.0000	1.0000	.9999	.9997	.9989	.9958	.9856	.9544	.8649	.6226
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0013	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	4	.0059	.0015	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	5	.0207	.0064	.0016	.0003	.0000	.0000	.0000	.0000	.0000	.0000
	6	.0577	.0214	.0065	.0015	.0003	.0000	.0000	.0000	.0000	.0000
	7	.1316	.0580	.0210	.0060	.0013	.0002	.0000	.0000	.0000	.0000
	8	.2517	.1308	.0565	.0196	.0051	.0009	.0001	.0000	.0000	.0000
	9	.4119	.2493	.1275	.0532	.0171	.0039	.0006	.0000	.0000	.0000
	10	.5881	.4086	.2447	.1218	.0480	.0139	.0026	.0002	.0000	.0000
	11	.7483	.5857	.4044	.2376	.1133	.0409	.0100	.0013	.0001	.0000
	12	.8684	.7480	.5841	.3990	.2277	.1018	.0321	.0059	.0004	.0000
	13	.9423	.8701	.7500	.5834	.3920	.2142	.0867	.0219	.0024	.0000
	14	.9793	.9447	.8744	.7546	.5836	.3828	.1958	.0673	.0113	.0003
	15	.9941	.9811	.9490	.8818	.7625	.5852	.3704	.1702	.0432	.0026
	16	.9987	.9951	.9840	.9556	.8929	.7748	.5886	.3523	.1330	.0159
	17	.9998	.9991	.9964	.9879	.9645	.9087	.7939	.5951	.3231	.0755
	18	1.0000	.9999	.9995	.9979	.9924	.9757	.9308	.8244	.6083	.2642
	19	1.0000	1.0000	1.0000	.9998	.9992	.9968	.9885	.9612	.8784	.6415
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A-6. CUMULATIVE NORMAL DISTRIBUTION (VALUES OF p CORRESPONDING TO Z_p FOR THE NORMAL CURVE)

Z_p	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5674	.4714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5967	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Source: After Pearson and Hartley, 1966. Used by permission of the Biometrika Trustees.

TABLE A-7. VALUES FOR j*, USED FOR ESTIMATING NONPARAMETRIC CONFIDENCE LIMITS FOR THE MEDIAN[1]

Sample Size (n)	j
5	5
6	6
7	7
8	7
9	8
10	9
11	9
12	10
13	10
14	11
15	12
16	12
17	13
18	13
19	14
20	15
21	15
22	16
23	16
24	17
25	18
26	18
27	19
28	19
29	20
30	20
31	21
32	22
33	22
34	23

TABLE A-7. (Continued)

Sample Size (n)	j
35	23
36	24
37	24
38	25
39	26
40	26
41	27
42	27
43	28
44	28
45	29
46	30
47	30
48	31
49	31
50	32

[1]In this table, j is the rank of the value used as an estimate of the nonparametric upper confidence limit of the median. Values shown are for two-sided confidence intervals with $\alpha = 0.05$.

From Van der Parren (1970).

Biometrika (1970), 57, 3, *p*. 613
Printed in Great Britain

Tables for distribution-free confidence limits for the median

BY J. L. VAN DER PARREN
Centre d'Etude de l'Energie Nucléaire: Studiecentrum voor Kernenergie, Belgium

SUMMARY

A table of the cumulative binomial distribution is used to provide confidence limits for the population median based on samples of size $n = 3(1)150$ from any continuous univariate distribution. The confidence interval covers the median with a probability of at least $1-2\alpha$, where $1-2\alpha = 0{\cdot}70$, $0{\cdot}80$, $0{\cdot}90$, $0{\cdot}95$, $0{\cdot}98$ and $0{\cdot}99$. The exact probability of coverage is given in each case.

1. INTRODUCTION

Thompson (1936) suggested a procedure for constructing a confidence interval for the median, independently of the underlying population form in the case of a continuous univariate distribution. Nair (1940) established a limited table using these order statistics. The present paper extends that of Nair.

Let $x_1, \ldots, x_n$ be the order statistics from a sample of size n. Let (x_k, x_{n-k+1}) be the smallest symmetrical interval of the form (x_i, x_{n-i+1}) covering the median with probability at least $1-2\alpha$, the exact probability being $1-2I$. Here k and I are tabulated as functions of n and $1-2\alpha$. In a few cases, the k-value retained corresponds to a probability slightly below $1-2\alpha$ though very close to it. An asterisk indicates this situation.

2. RELATED TOPICS

Relevant previous work includes the following. Geigy (1963) tabulates binomial confidence limits (x_g, x_d) for np, $p = 0{\cdot}5$ and $n = 6(1)1000$ at levels $2\alpha = 0{\cdot}05$ and $0{\cdot}01$. From these, the indices $(k, n-k+1)$ we are looking for are obtained as (x_g+1, x_d).

The subscript k in Table 1 can also be found as $(A+1)$, A being the critical value for the sign test in Owen (1962, Table 12.1). There

$$n = 1\ (1)\ 50\ (2)\ 100\ (10)\ 200\ (20)\ 500\ (50)\ 1000;$$

$$\alpha = 0{\cdot}005,\ 0{\cdot}01,\ 0{\cdot}025,\ 0{\cdot}05 \text{ and } 0{\cdot}10.$$

Critical values for the sign test are also tabulated by Dixon & Massey (1957, p. 417) for $n = 3\ (1)\ 90$ and $2\alpha = 0{\cdot}01$, $0{\cdot}05$, $0{\cdot}10$ and $0{\cdot}25$.

For values of the parameters n and α not appearing in Table 1 or in the above mentioned sources, the required information can be obtained alternatively from Tables of the Incomplete Beta-Function (Pearson, 1968) or from tables of cumulative binomial probabilities (Grubbs & Simon, 1952; Harvard University Press, 1955). In the last table,

$$n = 1\ (1)\ 50\ (2)\ 100\ (10)\ 200\ (20)\ 500\ (50)\ 1000.$$

Table 1. *Confidence limits for the median*

	$1-2\alpha$											
	0·70		0·80		0·90		0·95		0·98		0·99	
n	k	I	k	I	k	I	k	I	k	I	k	I
1												
2												
3	1	0·1250										
4	1	·0625	1	0·0625								
5	1	·0313	1	·0313	1	0·0313						
6	2	·1094	1	·0156	1	·0156	1	0·0156				
7	2	·0625	2	·0625	1	·0078	1	·0078	1	0·0078		
8	3	·1445	2	·0352	2	·0352	1	·0039	1	·0039	1	0·0039
9	3	·0898	3	·0898	2	·0195	2	·0195	1	·0020	1	·0020
10	3	·0547	3	·0547	2	·0107	2	·0107	1	·0010	1	·0010
11	4	·1133	3	·0327	3	·0327	2	·0059	2	·0059	1	·0005
12	4	·0730	4	·0730	3	·0193	3	·0193	2	·0032	2	·0032
13	5	·1334	4	·0461	4	·0461	3	·0112	2	·0017	2	·0017
14	5	·0898	5	·0898	4	·0287	3	·0065	3	·0065	2	·0009
15	5	·0592	5	·0592	4	·0176	4	·0176	3	·0037	3	·0037
16	6	·1050	5	·0384	5	·0384	4	·0106	3	·0021	3	·0021
17	6	·0717	6	·0717	5	·0245	5	·0245	4	·0064	3	·0012
18	7	·1189	6	·0481	6	·0481	5	·0154	4	·0038	4	·0038
19	7	·0835	7	·0835	6	·0318	5	·0096	5	·0096	4	·0022
20	8	·1316	7	·0576	6	·0207	6	·0207	5	·0059	4	·0013
21	8	·0946	8	·0946	7	·0392	6	·0133	5	·0036	5	·0036
22	9	·1431	8	·0669	7	·0262	6	·0085	6	·0085	5	·0022
23	9	·1050	8	·0466	8	·0466	7	·0173	6	·0053	5	·0013
24	9	·0758	9	·0758	8	·0320	7	·0113	6	·0033	6	·0033
25	10	·1148	9	·0539	8	·0216	8	·0216	7	·0073	6	·0020
26	10	·0843	10	·0843	9	·0378	8	·0145	7	·0047	7	·0047
27	11	·1239	10	·0610	9	·0261	8	·0096	8	·0096	7	·0030
28	11	·0925	11	·0925	10	·0436	9	·0178	8	·0063	7	·0019
29	12	·1325	11	·0680	10	·0307	9	·0121	8	·0041	8	·0041
30	12	·1002	11	·0494	11	·0494	10	·0214	9	·0081	8	·0026
31	13	·1405	12	·0748	11	·0354	10	·0147	9	·0053	8	·0017
32	13	·1077	12	·0551	11	·0251	10	·0100	10	·0100	9	·0035
33	14	·1481	13	·0814	12	·0401	11	·0175	10	·0068	9	·0023
34	14	·1147	13	·0607	12	·0288	11	·0122	10	·0045	10	·0045
35	14	·0877	14	·0877	13	·0448	12	·0205	11	·0083	10	·0030
36	15	·1215	14	·0662	13	·0326	12	·0144	11	·0057	10	·0020
37	15	·0939	15	·0939	14	·0494	13	·0235	12	·0100	11	·0038
38	16	·1279	15	·0716	14	·0365	13	·0168	12	·0069	11	·0025
39	16	·0998	16	·0998	14	·0266	13	·0119	12	·0047	12	·0047
40	17	·1341	16	·0769	15	·0403	14	·0192	13	·0083	12	·0032
41	17	·1055	16	·0586	15	·0298	14	·0138	13	·0057	12	·0022
42	18	·1400	17	·0821	16	·0442	15	·0218	14	·0097	13	·0040
43	18	·1110	17	·0631	16	·0330	15	·0158	14	·0069	13	·0027
44	19	·1456	18	·0871	17	·0481	16	·0244	14	·0048	14	·0048
45	19	·1163	18	·0676	17	·0362	16	·0178	15	·0080	14	·0033
46	19	·0920	19	·0920	17	·0270	16	·0129	15	·0057	14	·0023
47	20	·1214	19	·0719	18	·0395	17	·0200	16	·0093	15	·0040
48	20	·0967	20	·0967	18	·0297	17	·0147	16	·0066	15	·0028
49	21	·1264	20	·0762	19	·0427	18	·0222	16	·0047	16	·0047
50	21	·1013	20	·0595	19	·0325	18	·0164	17	·0077	16	·0033

Sample size n. Interval (x_k, x_{n-k+1}) has confidence coefficient $1-2I$ and is narrowest interval with confidence coefficient at least $1-2\alpha$. An asterisk denotes that I very slightly exceeds α.

Table 1 (*cont.*)

$1-2\alpha$

	0·70		0·80		0·90		0·95		0·98		0·99	
n	k	I	k	I	k	I	k	I	k	I	k	I
51	22	0·1312	21	0·0804	20	0·0460	19	0·0244	17	0·0055	16	0·0023
52	22	·1058	21	·0632	20	·0352	19	·0182	18	·0088	17	·0039
53	23	·1358	22	·0845	21	·0492	19	·0135	18	·0063	17	·0027
54	23	·1101	22	·0668	21	·0380	20	·0201	19	·0099	18	·0045
55	24	·1403	23	·0885	21	·0290	20	·0150	19	·0072	18	·0032
56	24	·1144	23	·0704	22	·0407	21	·0220	19	·0052	18	·0023
57	25	·1446	24	·0924	22	·0314	21	·0166	20	·0082	19	·0038
58	25	·1185	24	·0740	23	·0435	22	·0240	20	·0060	19	·0027
59	26	·1488	25	·0963	23	·0337	22	·0182	21	·0092	20	·0043
60	26	·1225	25	·0775	24	·0462	22	·0137	21	·0067	20	·0031
61	26	·1000	25	·0619	24	·0361	23	·0198	21	·0049	21	·0049
62	27	·1264	26	·0809	25	·0490	23	·0150	22	·0075	21	·0036
63	27	·1037	26	·0649	25	·0385	24	·0215	22	·0056	21	·0026
64	28	·1302	27	·0843	25	·0300	24	·0164	23	·0084	22	·0041
65	28	·1073	27	·0680	26	·0408	25	·0232	23	·0062	22	·0030
66	29	·1339	28	·0876	26	·0320	25	·0178	24	·0093	23	·0046
67	29	·1108	28	·0710	27	·0432	26	·0249	24	·0070	23	·0034
68	30	·1375	29	·0909	27	·0341	26	·0192	24	·0052	23	·0025
69	30	·1142	29	·0740	28	·0456	26	·0147	25	·0077	24	·0038
70	31	·1410	30	·0941	28	·0361	27	·0207	25	·0058	24	·0028
71	31	·1175	30	·0769	29	·0480	27	·0160	26	·0085	25	·0043
72	32	·1444	31	·0972	29	·0382	28	·0222	26	·0064	25	·0032
73	32	·1208	31	·0798	29	·0302	28	·0172	27	·0093	26	·0048
74	33	·1477	31	·0651	30	·0403	29	·0237	27	·0070	26	·0035
75	33	·1240	32	·0827	30	·0320	29	·0185	27	·0053	26	·0026
76	33	·1034	32	·0677	31	·0423	29	·0143	28	·0077	27	·0040
77	34	·1271	33	·0855	31	·0338	30	·0198	28	·0058	27	·0029
78	34	·1063	33	·0703	32	·0444	30	·0154	29	·0084	28	·0044
79	35	·1302	34	·0883	32	·0356	31	·0211	29	·0064	28	·0033
80	35	·1092	34	·0728	33	·0465	31	·0165	30	·0091	29	·0048
81	36	·1332	35	·0910	33	·0374	32	·0224	30	·0070	29	·0036
82	36	·1121	35	·0753	34	·0485	32	·0176	31	·0099	29	·0027
83	37	·1361	36	·0937	34	·0392	33	·0238	31	·0076	30	·0040
84	37	·1149	36	·0778	34	·0315	33	·0188	31	·0088	30	·0030
85	38	·1390	37	·0964	35	·0410	33	·0147	32	·0082	31	·0044
86	38	·1177	37	·0803	35	·0331	34	·0199	32	·0063	31	·0033
87	39	·1418	38	·0990	36	·0428	34	·0157	33	·0089	32	·0048
88	39	·1204	38	·0827	36	·0347	35	·0211	33	·0068	32	·0037
89	40	·1445	38	·0687	37	·0447	35	·0167	34	·0096	32	·0028
90	40	·1230	39	·0851	37	·0363	36	·0222	34	·0074	33	·0040
91	41	·1472	39	·0709	38	·0465	36	·0177	34	·0057	33	·0031
92	41	·1257	40	·0875	38	·0379	37	·0235	35	·0080	34	·0044
93	42	·1499	40	·0731	39	·0483	37	·0188	35	·0062	34	·0033
94	42	·1282	41	·0898	39	·0395	38	·0247	36	·0086	35	·0048
95	42	·1090	41	·0752	40	·0501*	38	·0198	36	·0067	35	·0037
96	43	·1307	42	·0921	40	·0411	38	·0158	37	·0092	35	·0028
97	43	·1114	42	·0774	40	·0335	39	·0209	37	·0072	36	·0040
98	44	·1332	43	·0944	41	·0427	39	·0167	38	·0098	36	·0030
99	44	·1138	43	·0795	41	·0349	40	·0219	38	·0077	37	·0043
100	45	·1356	44	·0967	42	·0443	40	·0176	38	·0060	37	·0033

Sample size n. Interval (x_k, x_{n-k+1}) has confidence coefficient $1-2I$ and is narrowest interval with confidence coefficient at least $1-2\alpha$. An asterisk denotes that I very slightly exceeds α.

Table 1 (*cont.*)

	$1-2\alpha$											
	0·70		0·80		0·90		0·95		0·98		0·99	
n	k	I	k	I	k	I	k	I	k	I	k	I
101	45	0·1161	44	0·0816	42	0·0364	41	0·0230	39	0·0082	38	0·0047
102	46	·1380	45	·0989	43	·0459	41	·0185	39	·0065	38	·0036
103	46	·1184	45	·0837	43	·0378	42	·0241	40	·0088	39	·0050*
104	47	·1403	45	·0705	44	·0475	42	·0195	40	·0069	39	·0039
105	47	·1207	46	·0857	44	·0392	42	·0157	41	·0094	39	·0030
106	48	·1426	46	·0724	45	·0491	43	·0204	41	·0074	40	·0042
107	48	·1229	47	·0878	45	·0407	43	·0165	42	·0099	40	·0033
108	49	·1449	47	·0743	45	·0335	44	·0214	42	·0079	41	·0045
109	49	·1251	48	·0898	46	·0421	44	·0173	42	·0062	41	·0035
110	50	·1471	48	·0762	46	·0348	45	·0224	43	·0084	42	·0049
111	50	·1273	49	·0918	47	·0435	45	·0182	43	·0066	42	·0038
112	51	·1493	49	·0780	47	·0361	46	·0234	44	·0089	42	·0029
113	51	·1294	50	·0938	48	·0450	46	·0190	44	·0070	43	·0041
114	51	·1116	50	·0799	48	·0373	47	·0243	45	·0094	43	·0032
115	52	·1315	51	·0957	49	·0464	47	·0199	45	·0075	44	·0044
116	52	·1136	51	·0817	49	·0386	47	·0161	46	·0099	44	·0034
117	53	·1336	52	·0977	50	·0478	48	·0208	46	·0079	45	·0047
118	53	·1156	52	·0835	50	·0399	48	·0169	46	·0063	45	·0037
119	54	·1356	53	·0996	51	·0493	49	·0216	47	·0084	46	·0050
120	54	·1176	53	·0853	51	·0412	49	·0177	47	·0067	46	·0039
121	55	·1376	53	·0727	51	·0343	50	·0225	48	·0089	46	·0031
122	55	·1195	54	·0871	52	·0425	50	·0184	48	·0071	47	·0042
123	56	·1396	54	·0744	52	·0354	51	·0234	49	·0093	47	·0033
124	56	·1215	55	·0889	53	·0438	51	·0192	49	·0075	48	·0045
125	57	·1415	55	·0761	53	·0366	52	·0243	50	·0098	48	·0035
126	57	·1233	56	·0906	54	·0451	52	·0200	50	·0079	49	·0048
127	58	·1434	56	·0777	54	·0377	52	·0163	50	·0063	49	·0037
128	58	·1252	57	·0923	55	·0463	53	·0208	51	·0083	50	·0050*
129	59	·1453	57	·0793	55	·0389	53	·0170	51	·0067	50	·0040
130	59	·1271	58	·0940	56	·0478	54	·0216	52	·0088	50	·0032
131	60	·1472	58	·0809	56	·0401	54	·0178	52	·0070	51	·0043
132	60	·1289	59	·0957	57	·0489	55	·0224	53	·0092	51	·0034
133	61	·1490	59	·0825	57	·0412	55	·0185	53	·0074	52	·0045
134	61	·1307	60	·0974	57	·0346	56	·0233	54	·0097	52	·0036
135	61	·1140	60	·0841	58	·0424	56	·0192	54	·0078	53	·0048
136	62	·1324	61	·0991	58	·0357	57	·0241	54	·0063	53	·0038
137	62	·1158	61	·0857	59	·0436	57	·0200	55	·0082	53	·0030
138	63	·1342	61	·0738	59	·0367	58	·0249	55	·0066	54	·0040
139	63	·1175	62	·0873	60	·0447	58	·0207	56	·0086	54	·0032
140	64	·1359	62	·0753	60	·0378	58	·0171	56	·0070	55	·0043
141	64	·1191	63	·0888	61	·0459	59	·0214	57	·0090	55	·0034
142	65	·1376	63	·0767	61	·0388	59	·0178	57	·0073	56	·0045
143	65	·1208	64	·0903	62	·0470	60	·0222	58	·0094	56	·0036
144	66	·1393	64	·0782	62	·0399	60	·0184	58	·0077	57	·0048
145	66	·1224	65	·0918	63	·0482	61	·0229	59	·0099	57	·0038
146	67	·1410	65	·0796	63	·0409	61	·0191	59	·0080	58	·0050*
147	67	·1241	66	·0934	64	·0493	62	·0237	59	·0065	58	·0040
148	68	·1426	66	·0810	64	·0420	62	·0198	60	·0084	58	·0032
149	68	·1257	67	·0949	64	·0356	63	·0245	60	·0069	59	·0043
150	69	·1442	67	·0824	65	·0430	63	·0204	61	·0088	59	·0034

Sample size n. Interval (x_k, x_{n-k+1}) has confidence coefficient $1-2I$ and is narrowest interval with confidence coefficient at least $1-2\alpha$. An asterisk denotes that I very slightly exceeds α.

3. Basic equations and procedure

Let $z_1, \ldots, z_n$ be a random sample from a continuous population with density function $f(z)$ and

$$F_i = F(x_i) = \int_{-\infty}^{x_i} f(z)\,dz$$

be the area of $f(z)$ which is less than the ith ordered observation x_i.

Let $G_{k,s}$ be the distribution function of the joint distribution of F_k and F_s $(k < s)$. The interval (x_k, x_s) covers the p-quantile X_p whenever $F_k \leqslant p \leqslant F_s$. This happens with probability

$$1-2I = \int_0^p \int_p^1 dG_{k,s}(F_k, F_s).$$

This can be shown (Kendall & Stuart, 1969, p. 518) to be equivalent to

$$1-2I = I_p(k, n-k+1) - I_p(s, n-s+1),$$

where $I_p(i,j)$ is the incomplete beta function.

If the quantile considered is the median, this yields

$$1-2I = I_{0.5}(k, n-k+1) - I_{0.5}(s, n-s+1),$$

when $k+s = n+1$, we obtain a similar situation for ascending and decreasing order, i.e.

$$1-2I = 1-2I_{0.5}(k, n-k+1)$$
$$= \sum_{r=k}^{n-k} \binom{n}{r} p^r q^{n-r} \quad (p = q = \tfrac{1}{2}).$$

Grubbs & Simon (1952) tabulate

$$\sum_{r=c}^{n} \binom{n}{r} p^r (1-p)^{n-r}.$$

Therefore k can easily be obtained as the largest integer for which

$$\sum_{r=k}^{n} \binom{n}{r} p^r (1-p)^{n-r} \geqslant 1-\alpha.$$

It is a pleasure to thank Mrs Jonckers, who helped in constructing the tables and typed the text, and to Mrs Weyers who typed the tables.

References

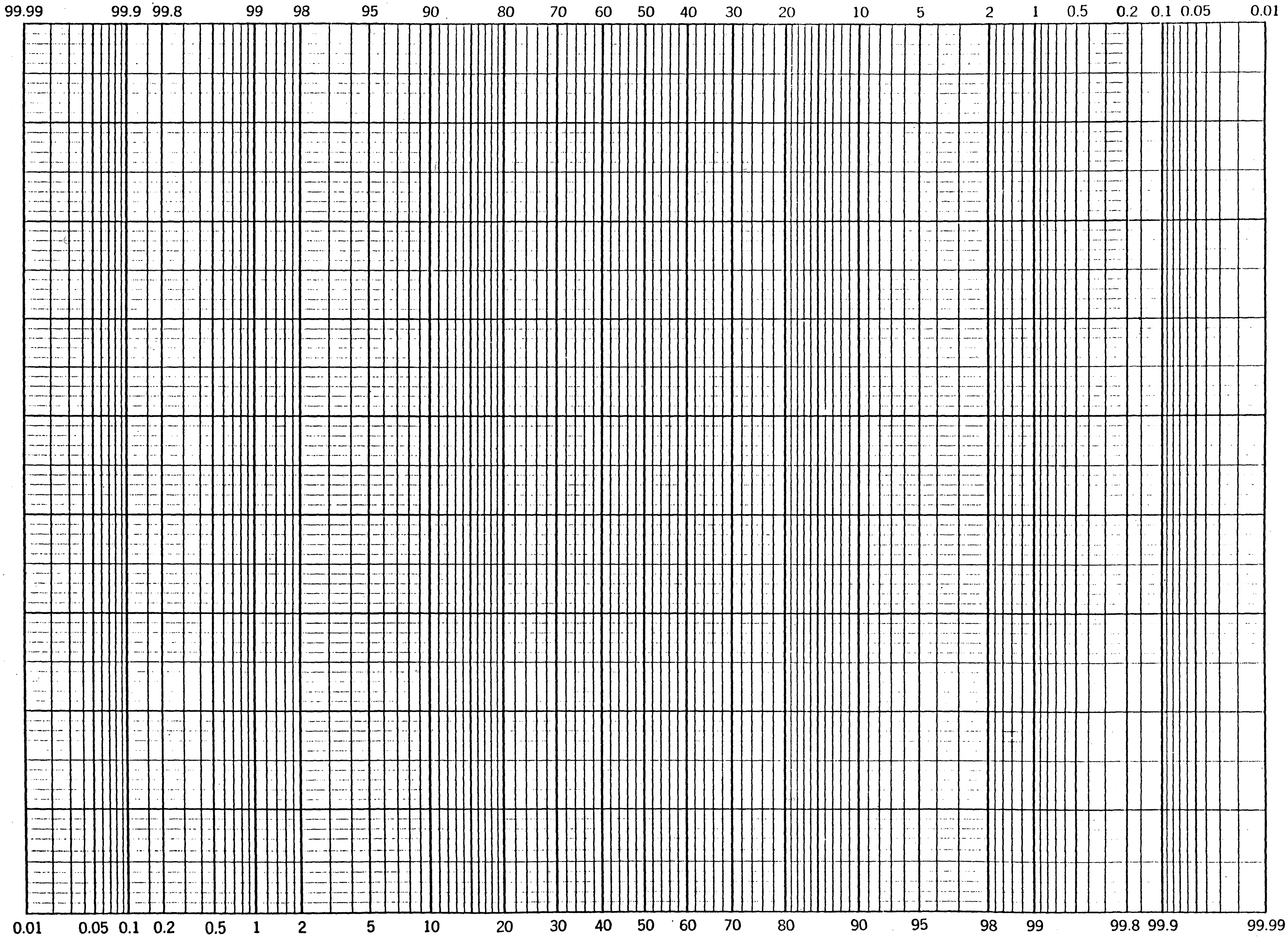
Dixon, W. J. & Massey, F. J. (1957). *Introduction to Statistical Analysis*. New York: McGraw Hill.
Geigy, J. R.–S. A. (Editor.) (1963). *Tables Scientifiques*. Documenta Geigy (Bâle).
Grubbs, L. E. & Simon, F. E. (1952). Tables of the cumulative binomial probabilities. Department of the Army—Office of Chief Ordnance—Washington 25 D.C. USA.
Harvard University Press (1955). *Tables of the Cumulative Binomial Sum*. Cambridge, Mass.: Harvard University Press.
Kendall, M. G. & Stuart, A. (1969). *The Advanced Theory of Statistics*, vol. II, 3rd edition. London: Griffin.
Nair, K. R. (1940). Table of confidence intervals for the median in samples from any continuous population. *Sankhyā*, **4**, 551–8.
Owen, D. B. (1962). *Handbook of Statistical Tables*. London: Pergamon.
Pearson, K. (1968). *Tables of the Incomplete Beta-Function*, 2nd edition. Cambridge University Press.
Thompson, W. R. (1936). On confidence ranges for the median and other expectation distributions for populations of unknown distribution form. *Ann. Math. Statist.* **7**, 122–8.

[*Received February* 1970. *Revised May* 1970]

Editorial note added in proof. Reference should also be made to the extensive tables of W. J. MacKinnon (*J. Am. Statist. Ass.* **59**, 1964, 935–56), who, however, did not give I.

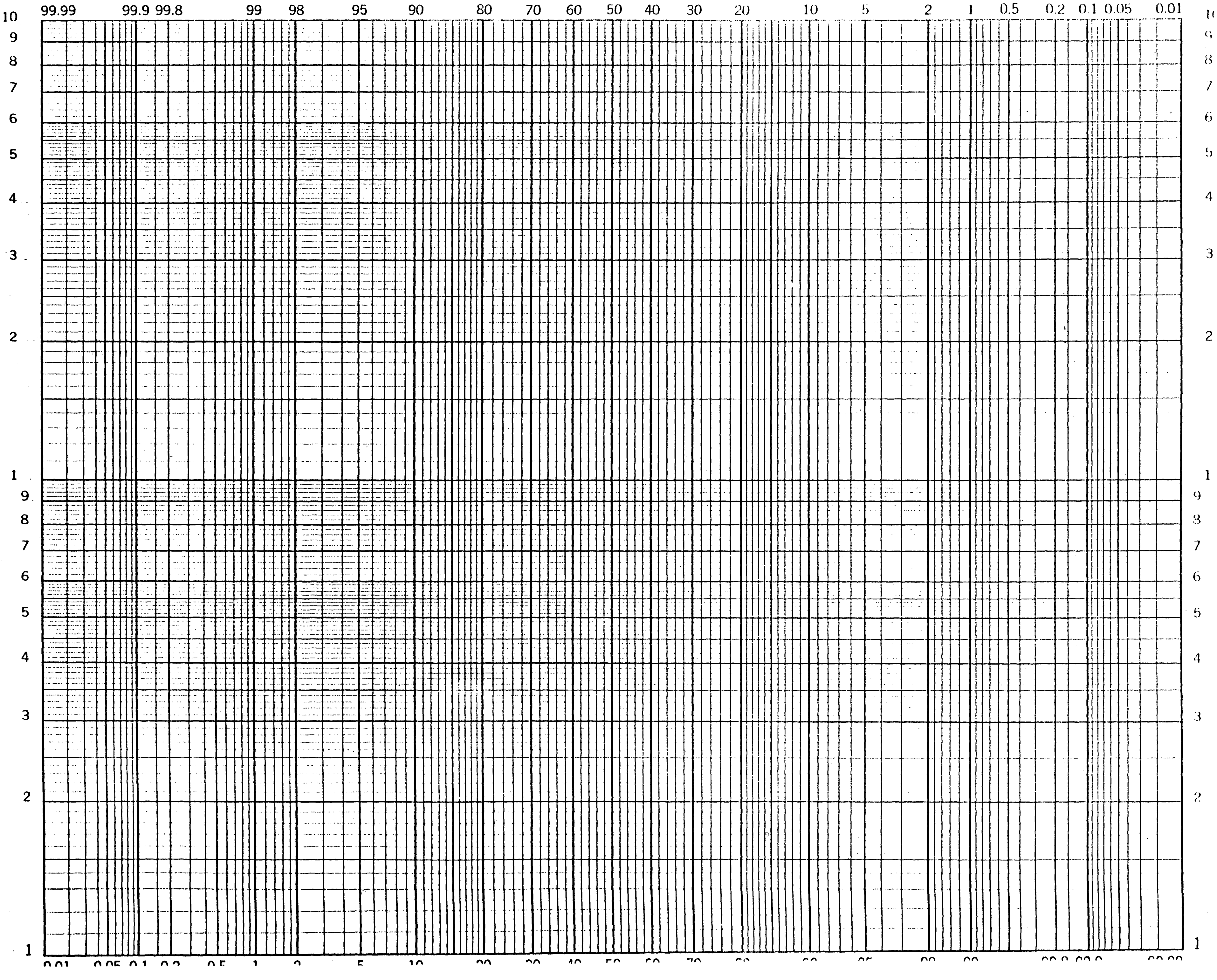
K+E PROBABILITY X 90 DIVISIONS
KEUFFEL & ESSER CO. MADE IN U.S.A.

46 8000



K+E PROBABILITY X 2 LOG CYCLES
KEUFFEL & ESSER CO. MADE IN U.S.A.

46 8040



SUPPLEMENTS

Note: Sections, Figures and Examples cited here are found in the main Guidance Document.

SUPPLEMENT S-1

SUMMARY TABLE - Calculating a Value for Comparison with a Cleanup Standard.

SITE DATA DISTRIBUTION		TOXIC EFFECTS	SOIL	GROUNDWATER
Lognormal (default)		Chronic	UCL calculated using H-statistic (5.2.1.2) See: Worksheet W-2, Example 11 (under Criterion 1)	Same as for soil (5.2.1.2)
		Short Term/Acute	UTL on 90th percentile using $\log_e$-transformed data (5.2.2.2)	UTL on 50th percentile (median) (5.3.2)
Normal		Chronic	UCL on mean (5.2.1.1). See Example 13.	Same as for soil (5.2.1.1)
		Short Term/Acute	UTL on 90th percentile (5.2.2.1). See Example 14.	UCL on mean (5.3.1, 5.2.1.1). See Example 13.
Significantly different from lognormal and normal	N≤20	Chronic	• Consider obtaining additional samples Requires consultation with Ecology: • UTL on site-specific percentile (5.2.2.3) See Example 15. • UCL using Z statistic? (5.2.1.3) See Example 16.	Same as for soil
		Short Term/Acute	• Consider obtaining additional samples • UTL on 90th percentile (5.2.2.3) See Example 15.	• Consider obtaining additional samples • UTL on median (5.3.3) See Example 17.
	N>20	Chronic	Requires consultation with Ecology: • UCL on site-specific percentile (5.2.2.4) • UCL using Z statistic? (5.2.1.3)	Same as for soil
		Short Term/Acute	UCL on 90th percentile (5.2.2.4)	UCL on median (5.3.3)

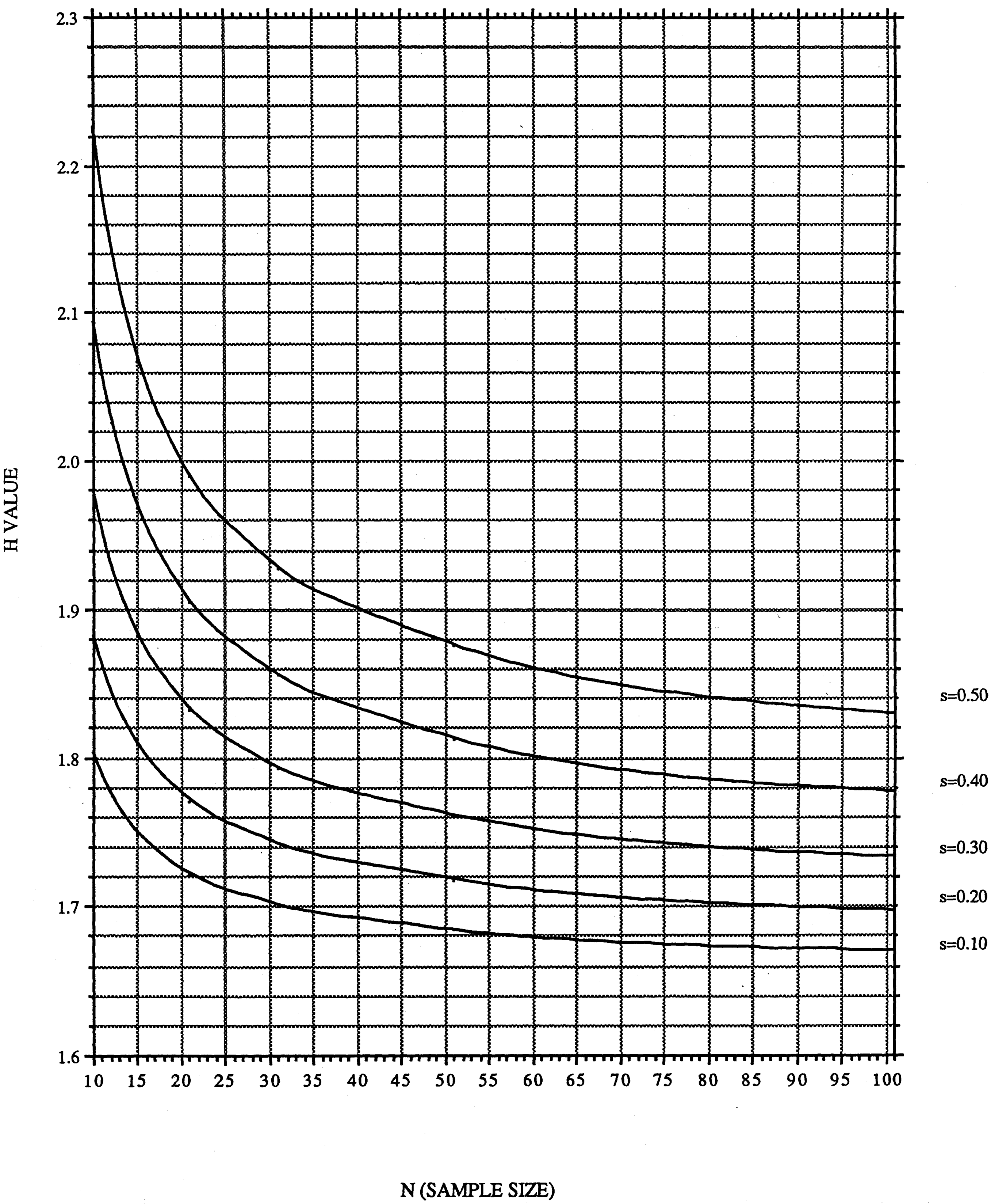
Abbreviations UCL: Upper confidence limit UTL: Upper tolerance limit

ADDITIONAL REQUIREMENTS: When Cleanup Standard is not based on background, other compliance criteria are: (1) No single value more than twice the Cleanup Standard, and (2) No more than 10% of values greater than Cleanup Standard. When Cleanup Standard is based on background, different criteria may apply (Figure 12, Section 4.3.3.2).

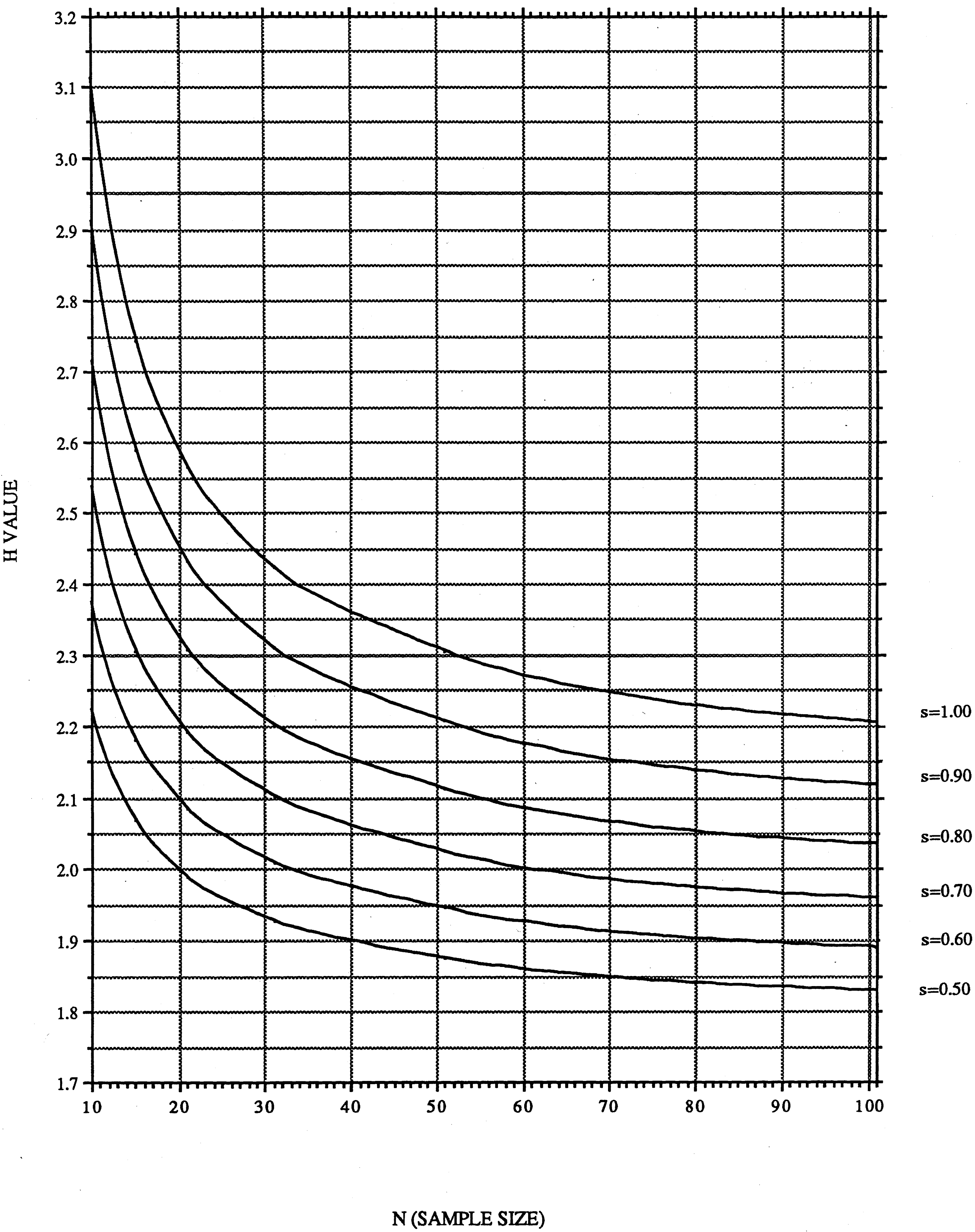
SUPPLEMENT S-2

Additional graphs for estimating H-values to calculate upper 95% confidence limits (lognormal distribution). Supplement to Figure A-1.

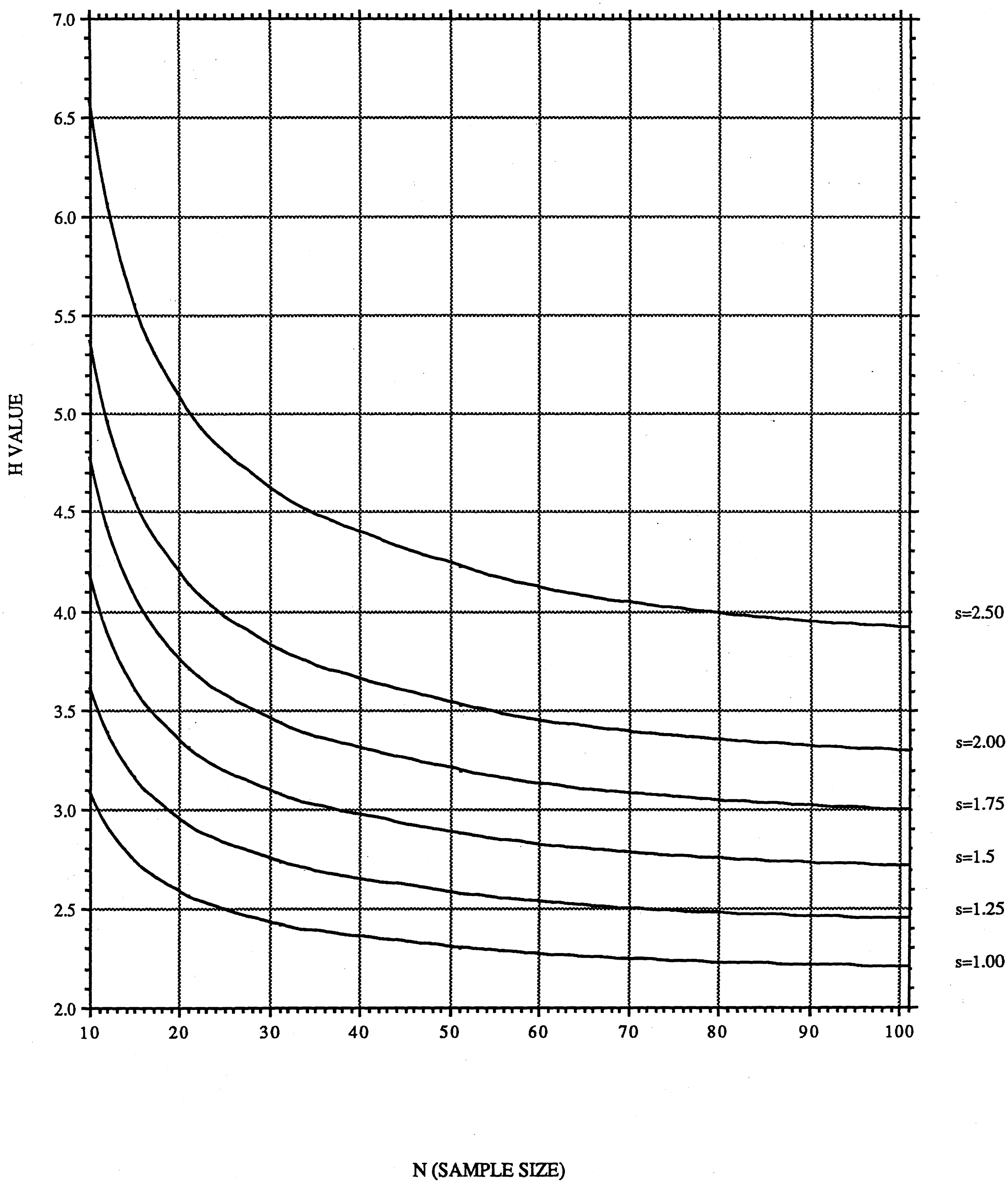
H Values for Std. Devn. = 0.10 to 0.50 (Std. Devn. of log-transformed data)



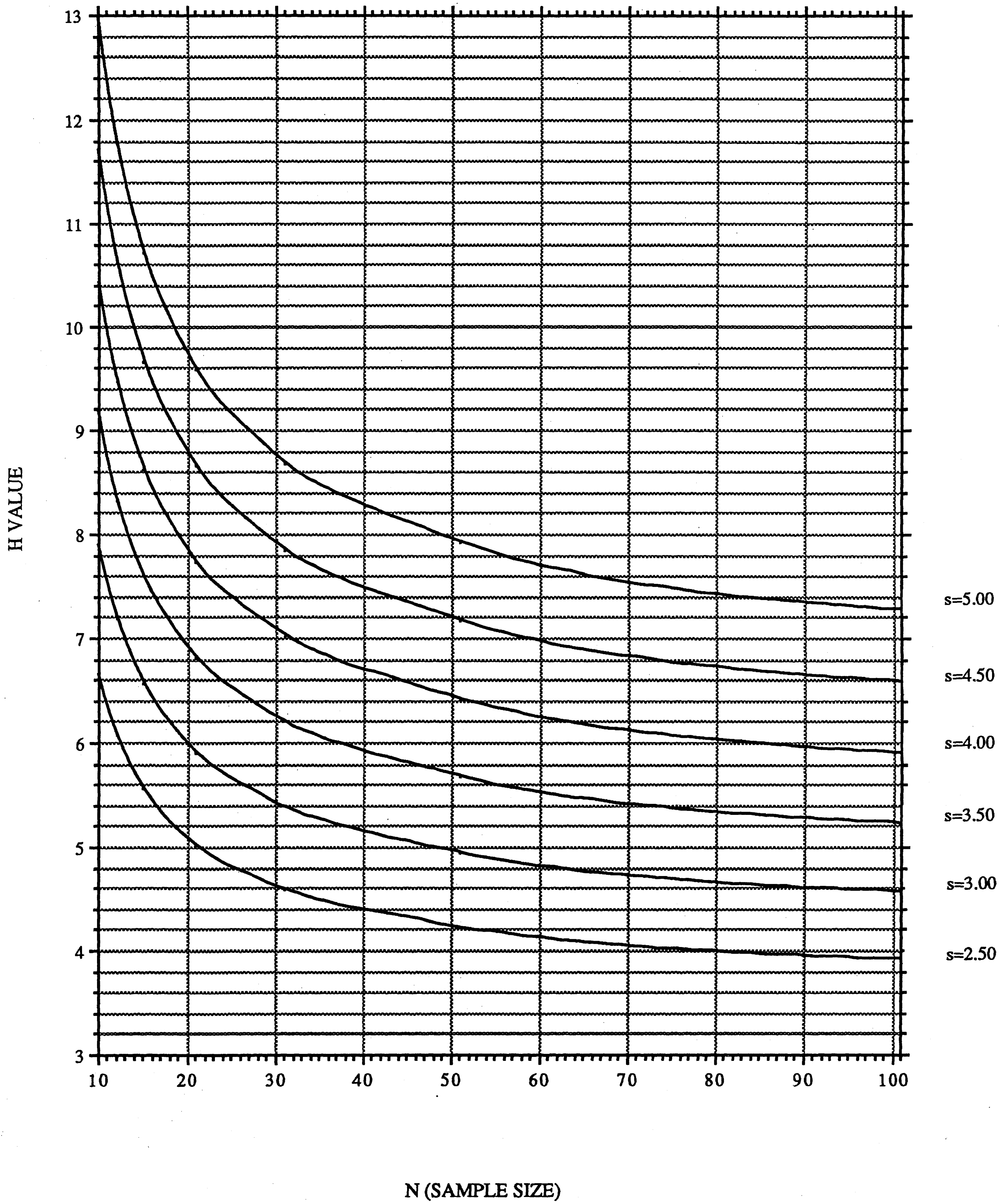
H Values for Std. Devn. = 0.50 to 1.00 (Std. Devn. of log-transformed data)



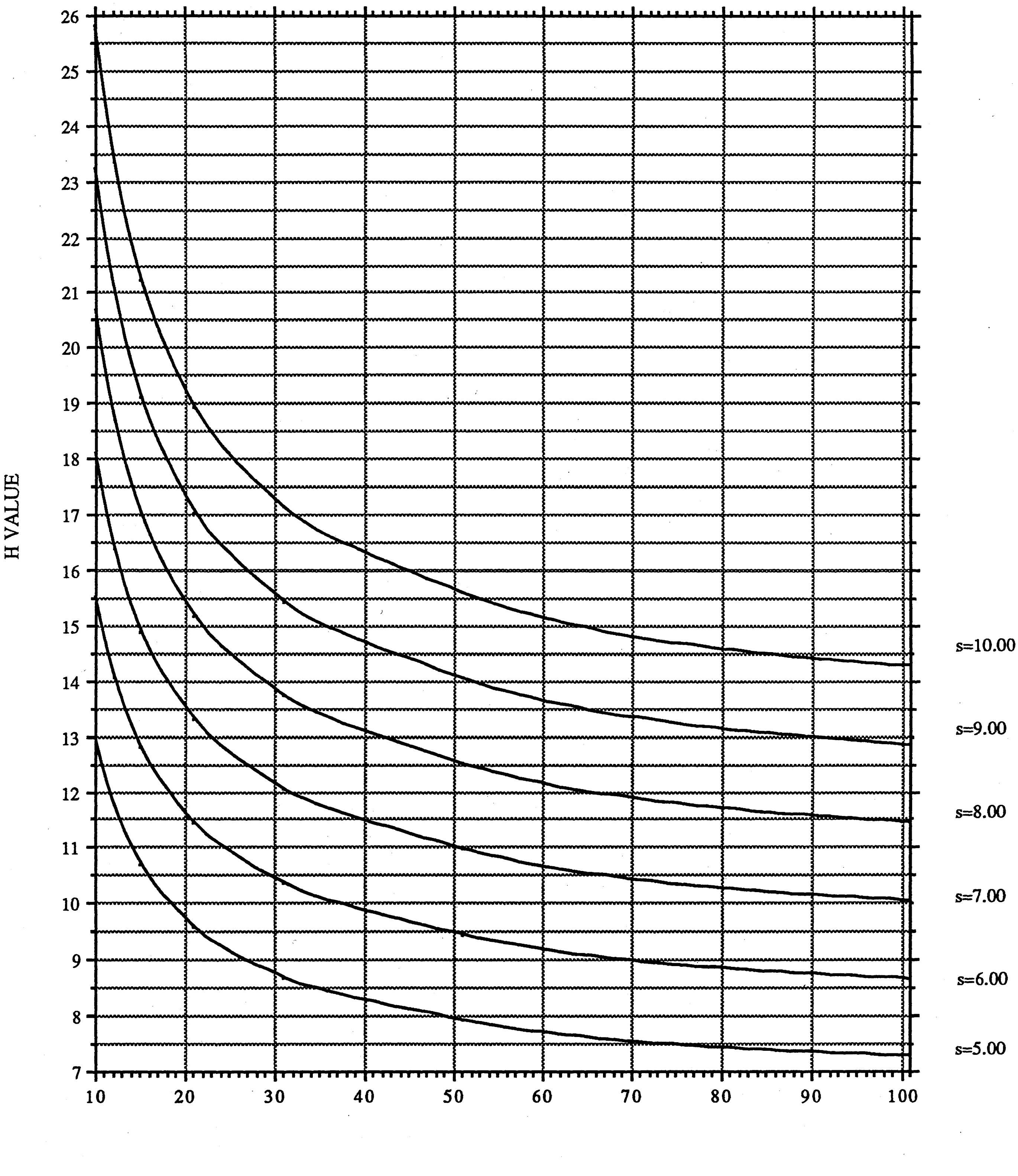
H Values for Std. Devn. = 1.00 to 2.50 (Std. Devn. of log-transformed data)



H Values for Std. Devn. = 2.50 to 5.00 (Std. Devn. of log-transformed data)



H Values for Std. Devn. = 5.00 to 10.00 (Std. Devn. of log-transformed data)



SUPPLEMENT S-3

PROCEDURE FOR DETERMINING THE DISTRIBUTION OF SITE OR BACKGROUND DATA (See Section 2.1.4)

Step 1. Default assumption is lognormal distribution.

Step 2. Look at a probability plot of the log-transformed data to check the assumption of a lognormal distribution (see Section 2.1.4.2). The procedure is the same as shown in Example 4, except that the data must be log-transformed first. Probability paper is supplied at the end of the Guidance Document.

Although Example 4 shows how to do the plots by hand, it is easier to use statistical software such as STATGRAPHICS® or SYSTAT® for this purpose.

> STATGRAPHICS® Log-transform the data first. Then, at the main menu, select option **G. Estimation and Testing** under **PLOTTING AND DESCRIPTIVE STATISTICS**. Next, select option **3. Normal Probability Plot** in the **ESTIMATION AND TESTING** menu. At the **Data vector** field prompt, enter the variable name used for the log-transformed data.

The points should lie on a straight line if the data are lognormally distributed. This is a subjective test. The fit to a straight line should be "good" but need not be exact (compare Figures 6 and 7).

If the fit is poor or questionable, proceed to Step 3. Otherwise, conclude that a lognormal distribution is valid, and stop here. [Note: Step 2 is provided for convenience but it is acceptable to proceed directly from Step 1 to Step 3.]

Step 3. Test for lognormality using the W test. Log-transform the data before doing the test. (See Example 7 and Worksheet W-1.) For sample sizes greater than 50, D'Agostino's test should be used (see section 2.1.4.1).

If the test does not indicate a significant difference from lognormality, conclude that a lognormal distribution is valid, and stop here. If lognormality is rejected, go to Step 4.

Step 4. Repeat the procedures described in Step 2, but *do not log-transform the data*. If the fit to a straight line is poor or questionable, proceed to Step 5. Otherwise, conclude that a normal distribution is valid, and stop here. [Note: Step 4 is for convenience and is optional; alternatively, go directly from Step 3 to Step 5.]

Step 5. Test for normality using the W test (Example 7, Worksheet W-1a). For sample sizes greater than 50, D'Agostino's test should be used (see Section 2.1.4.1).

If the test does not indicate a significant difference from normality, conclude that a normal distribution is valid. If normality is rejected, conclude that the data are significantly different from both lognormal and normal distributions and follow instructions for "non-lognormal, non-normal" distribution data in the Guidance Document.

SUPPLEMENT S-4

PROCEDURE FOR CALCULATING BACKGROUND VALUE.

See Figure 12 for complete flowchart and Section 4.3.3.2

PART I (See Example 12. Although groundwater is used in the example, calculations are the same for soil).

Step 1. Is the default assumption that background data are lognormally distributed rejected? (See Supplement S-3.) If data are not lognormally distributed, go to PART II. Otherwise, proceed to Step 2.

Step 2. Lognormal distribution. Calculate the 90th percentile value. (See Section 2.1.2.2, Example 10 and Worksheet W-3.)

Step 3. Calculate the 50th percentile (median). Several methods can be used (e.g. Section 2.1.2.1, Examples 3, 4 and 12). Worksheet W-3 uses the method from Example 12.

Step 4. Is the 90th percentile more than 4 times the 50th percentile?

NO: Use the 90th percentile as the background value

YES: Use 4 times the 50th percentile value

Note: Ecology may require a different percentile if background data are lognormally distributed but compliance monitoring (site) data are not (Section 4.3.3.2, Figure 12).

PART II Data are not lognormally distributed.

Step 1. Is the alternative that data are normally distributed rejected? (See Supplement S-2.) If data are neither lognormally or normally distributed, go to PART III. Otherwise, proceed to Step 2.

Step 2. Normal distribution. Use a percentile as the background value.
In general, Ecology may use the 80th percentile (Example 9) as the background value. However, the department may also determine that another percentile is more appropriate on a site-specific basis.

PART III Data are neither normally or lognormally distributed.

Requires site-specific decision by Ecology. Options include:

(1) Percentile calculated using nonparametric methods (see Examples 5 and 9)

(2) Wilcoxon rank sum test (Mann Whitney U test).

SUPPLEMENT S-5

PROCEDURE FOR CALCULATING COEFFICIENT OF VARIATION FROM BEST-FIT DISTRIBUTION.

Decisions relating to the coefficient of variation of a contaminant distribution (e.g. see Section 4.3.5) should be based on the best-fit distribution, not the sample statistics (see Section 2.1.3 and Example 12). This supplement describes a simple procedure for calculating the coefficient of variation using the statistical software package, STATGRAPHICS®. It is assumed here that a decision has already been made regarding the data distribution (Supplement S-3). This information is needed in Step 4.

Step 1. Under the **PLOTTING AND DESCRIPTIVE STATISTICS** submenu, select option **H (Distribution Functions)**.

Step 2. In the **DISTRIBUTION FUNCTIONS** menu, select option **1 (Distribution Fitting)**.

Step 3. The **Distribution Fitting** menu appears next. Data must be provided in the **Data vector** field. There are two ways to do this:

1) Enter the data in the **Data vector** field from the keyboard. Use a comma or space to separate different values.

2) Enter the name of a file and variable if the data have already been entered in a STATGRAPHICS® file. For help, press F7, then scroll through the list. When you find the variable, press Enter.

Step 4. Enter the appropriate distribution number from the menu in the next field. For example, enter 13 for the lognormal distribution.

Step 5. Press F6. The best-fit values for the mean and standard deviation will then be displayed. Divide the standard deviation by the mean to get the coefficient of variation (CV).

For further information: To compare the best-fit distribution with the data distribution, press F6 again and select Histogram from the next menu. The screen will then show options for the graph. After making any desired changes, press F6 to see the plot.

Statistical Guidance for Ecology Site Managers

SUPPLEMENT S-6

ANALYZING SITE OR BACKGROUND DATA WITH BELOW-DETECTION LIMIT OR BELOW-PQL VALUES (CENSORED DATA SETS)

August, 1993

Definitions

An analysis of a sample that is reported as below the detection limit or below the practical quantitation limit (PQL) is referred to here as a **censored value**. **Censored data** means a data set that includes one or more censored values. The term therefore includes data sets that contain both censored and uncensored values. **Uncensored data** means a data set that consists entirely of uncensored values.

Introduction

As discussed in Section 2.2 of the *Statistical Guidance for Ecology Site Managers* ("Guidance"), the analysis of site or background data with censored values can be a difficult statistical problem. For example, the use of routine statistical procedures described in the Guidance to analyze censored data may lead to erroneous conclusions on site compliance with a cleanup standard.

- The primary purpose of this Supplement is to provide more detailed recommendations on acceptable methods for analyzing censored data sets than were previously given in the Guidance document.

- This Supplement also provides new guidance applicable to either uncensored or censored data on the use of normal probability plots to:

    - make quantitative, unambiguous decisions on the appropriate statistical distribution for the data. This replaces the need for a subjective decision on whether data points fit a straight line, as previously recommended in the Guidance.

    - find the value corresponding to a percentile (for background data) using least-squares linear regression. This avoids problems arising from the previously-recommended procedure of attempting to obtain a value from the probability plot by visual inspection.

    - calculate the coefficient of variation (for background data) from the best-fit distribution estimates for site or background parameters (μ and σ), rather than using previous methods in the Guidance which estimate sample statistics ($\bar{x}$ and s).

Ecology recognizes that other statistically defensible methods for analyzing censored data may exist. Alternative approaches which may be proposed on a site-specific basis may be approved if adequately supported (e.g., by including relevant material from the statistical literature). Some approaches described in this Supplement include the requirement to consult with Ecology. Refer to Guidance Section 1.2 regarding the applicability of this requirement. Ecology invites written comments on the methods described in this Supplement for consideration in evaluating the need for future revisions.

Which statistical distribution fits the data?

A decision on the appropriate distribution of the data (lognormal, normal or neither) must be made before proceeding with the analysis of site (compliance) or background data. The Guidance describes two methods for making this decision: the use of normal probability plots and the W test (replaced by D'Agostino's test for n >50).

For censored data, the normal probability plot method should be used. Use of the W test or D'Agostino's test requires simple substitution for censored values (e.g., replacing non-detects with half the detection limit), which can lead to erroneous conclusions on the appropriate distribution with these tests. The normal probability plot method described below uses the uncensored values in the data set to determine the appropriate distribution.

The procedures for using normal probability plots described in the Guidance rely on visual inspection of the plot for linearity and, in the case of background data, to estimate the value corresponding to a percentile. However, because of the uncertainties that arise from visual inspection, the procedure described below should be used to provide a quantitative, unambiguous decision on linearity and to calculate a percentile value. This procedure should also be used when evaluating data that do not contain censored values. It replaces an earlier procedure for calculating background values that was shown in Guidance Worksheet W-3.

Normal probability plot analyses

Distribution decision

Step 1.

List the values from lowest to highest and assign a rank to each. If there are censored values, assign half the detection limit to non-detects or the method detection limit to below-practical quantitation limit values. For example, the data set:

12.0
8.9
ND (detection limit = 10)
ND (detection limit = 8)

would be ranked:

Data	Rank
4	1
5	2
8.9	3
12.0	4

Normal probability plot analyses (cont.)

Step 2. The remaining calculations are performed only with the uncensored values. Assign a score to each uncensored value:

$$\text{score}_i = \Phi^{-1}[(i - 3/8) / (n + 0.25)]$$

where Φ^{-1} is the inverse of the standard normal cumulative distribution (obtained using Guidance Table A-6), i is the rank of the value, and a is the total number of data values (i.e. censored and uncensored).

Step 3.

Use the uncensored values and corresponding scores to calculate the least-squares linear regression equation and the correlation coefficient, r. If testing for a lognormal distribution, "uncensored values" means the log-transformed values. When testing for a normal distribution, use the untransformed values.

Decision criteria:

1) Do not proceed unless the regression ANOVA F-value is significant at the $p = 0.05$ level.

2) If the square of the correlation coefficient (r^2) for the analysis of the log-transformed values is 0.9 or higher, assume that the data are lognormally distributed. (An r^2 of 1 indicates a perfect fit to a straight line.)

If the lognormal distribution is rejected, then if r^2 for the analysis of the untransformed values is 0.9 or higher, assume that the data are normally distributed. Otherwise, reject the normal distribution.

These criteria should also be used for probability plot analyses of uncensored data. In most cases, the criteria are expected to lead to the same conclusion on the appropriate distribution as decisions based on the W test (or D'Agostino's test for $n > 50$) when applied in the same sequence. That is, the data are tested first for a lognormal distribution and if this assumption is rejected, then for a normal distribution. Where there is a discrepancy, either method (i.e. probability plot or one of the two tests) for analyzing uncensored data is acceptable to Ecology.

Background values

If a background value is to be calculated under the assumption of a lognormal or normal distribution, use the regression equation $\text{score}_1 = f(\text{value}_i)$ to calculate the value corresponding to a percentile. If the percentile is P, calculate the score as $\Phi^{-1}(P/100)$ and solve for the corresponding value. In the lognormal case, the regression is based on log-transformed data, and the solution must therefore be converted: Final value = exp(value from regression equation).

Normal probability plot analyses (cont.)

Coefficient of variation

Normal distribution.

Use the least-squares linear regression equation for the normal probability plot to obtain estimates for the mean (μ) and standard deviation (σ). Calculate the coefficient of variation (μ/σ).

The regression equation has the form:

$$y = mx + b$$

where y = scores (see Step 2 under Distribution decision)
m = slope of the regression line
x = data values
b = regression intercept

The mean is obtained by solving for y = 0, and the standard deviation by solving for y = 1 and subtracting the mean. Using elementary algebra, this leads to the following simple results:

Mean = -b/m

Standard deviation = 1/m

Lognormal distribution

The approach is the same as for the normal distribution, except that the linear regression is based on log-transformed values. This requires an additional step to estimate the untransformed mean and standard deviation, using equations 13.7 and 13.8 in Gilbert (1987). For computational purposes, the required statistics are calculated as follows:

$$\text{Mean} = \exp[(-b/m) + (1/2)(1/m^2)] = \mu$$

$$\text{Standard deviation} = \sqrt{\{\mu^2[\exp(1/m^2) - 1]\}}$$

Calculate the coefficient of variation from these statistics (standard deviation/mean).

Compliance decisions on site data: calculation of an upper 95% confidence limit (UCL) on the site mean.

Summary Table for Substances with Chronic Toxic Effects

Percentage of non-detects or below-PQL values in data set	Recommended procedure	Discussion
More than 0% but no more than 15%	Replace NDs with 1/2DL, and below-PQLs with MDL	Case 1
Between 15% and 50%	*Lognormal distribution:* Use Cohen's method-adjusted mean and standard deviation of log-transformed data to calculate UCL (Worksheets W-4 and W-2). *Normal distribution:* Use Cohen's method-adjusted mean and standard deviation of untransformed data to calculate UCL (Worksheet W-4a). *Neither distribution*: Use the largest value in the data set as the UCL.	Case 2
More than 50%	Use the largest value in the data set as the UCL.	Case 3

Case 1. No more than 15% of the data are censored values (non-detects or below-PQL values).

Note: The criterion of 15% is recommended in statistical guidance provided by EPA (U.S. EPA 1989, 1992).

PART I — Default assumption is that data come from a lognormal distribution.

Step 1. Test the default assumption of a lognormal distribution using the normal probability plot procedure described on page 2. If the assumption is rejected, proceed to Part II.

Step 2. Substitute half the detection limit for non-detects and the method detection limit for below-PQL values. Calculate the upper 95% confidence limit using Land's method (Worksheet W-2).

PART II — Data are not lognormally distributed.

Site data (cont.)

Step 1. Test the default assumption of a normal distribution using the normal probability plot procedure described on page 2. If the assumption is rejected, proceed to Part III.

Step 2. Substitute half the detection limit for non-detects and the method detection limit for below-PQL values. Calculate the upper 95% confidence limit using the t-statistic (see Section 5.2.1 of the Guidance).

PART III Data are neither lognormally nor normally distributed.

Use the maximum value in the data set as the upper 95% confidence limit. See page 8.

Notes: The basis for this recommendation is explained in Note 2 (attached).

Other approaches for cases where both the lognormal and normal distributions are rejected (Guidance Sections 5.2.13-5.2.1.4) may be applicable but should not be used for censored data without support from a qualified statistician.

Site data (cont.)

Case 2. More than 15% but not more than *50%* of the data are censored values.

PART I Default assumption is that data come from a lognormal distribution.

Step 1. Test the default assumption of a lognormal distribution using the normal probability plot procedure described on page 2. If the assumption is rejected, proceed to Part II.

Step 2. Calculate the adjusted mean and standard deviation, using the log-transformed data(and log-transformed detection limit or PQL), by Cohen's method. Worksheet W-4 is provided here for assistance.

About Cohen's method....

Cohen's method is a maximum likelihood estimation (MLE) procedure for adjusting the sample mean and standard deviation to account for data below the detection limit (or PQL) The data are assumed to be normally distributed. For the lognormal case, log-transformed values will be normally distributed. Further information on Cohen's method is given in Cohen (1959,1961), EPA (1989, 1992) and Gilbert (1987). Detailed computational instructions are given in EPA (1989, p. 8-7).

Cohen's method becomes less reliable with small sample sizes, a general problem with MLE methods (Helsel 1990). If results from the use of this method with a particular data set seem questionable, the option of increasing the sample size should be considered.

Step 3. Enter the adjusted mean and standard deviation in boxes M and S on Worksheet W-2 of the Guidance document. Complete the remaining calculations on that worksheet to calculate the upper 95% confidence limit. Include both censored and uncensored data in determining the number of samples (box N of Worksheet W-2).

PART II Data are not lognormally distributed.

Step 1. Test the default assumption of a normal distribution using the normal probability plot procedure described on page 2. If the assumption is rejected, proceed to Part III.

Step 2. Calculate the adjusted mean and standard deviation, using the raw data (and untransformed detection limit or PQL), by Cohen's method (see box). Worksheet W-4a is provided here for assistance.

Step 3. Use the adjusted mean and standard deviation to calculate the upper 95% confidence limit using the t-statistic (see Section 5.2.1 of the Guidance). Note: include both censored and uncensored data in determining the number of samples (n).

Site data (cont.)

Multiple detection limits

If the data set contains non-detects at different detection limits, use either of the following approaches for calculating an upper confidence limit:

1) If Cohen's method is used, a single detection limit for the data set is required for the calculations Use the highest of the detection limits reported for the non-detects in the data set.

2) Assign each non-detect a value equal to half the detection limit reported for that sample (simple substitution). Then use the routine statistical procedures for calculating an upper confidence limit under the lognormal or normal distribution assumptions. (That is, treat the data set as under Case 1 even if 15%-50% of the data are censored.)

The same recommendations apply to data censored at the PQL, except that the method detection limit is used in place of half the detection limit.

PART III Data are neither lognormally nor normally distributed.

Use the maximum value in the data set as the upper 95% confidence limit. See page 8.

Notes: The basis for this recommendation is explained in Note 2 (attached).
Other approaches for cases where both the lognormal and normal distributions are rejected (Guidance Sections 5.2.1.3-5.2.1.4) may be applicable but should not be used for censored data without support from a qualified statistician.

Case 3. More than 50% of the data are censored values.

Use the maximum value in the data set as the upper 95% confidence limit. See page 8.

Notes: The basis for this recommendation is explained in an attached technical note.
If a lognormal or normal distribution can be demonstrated, or if the default lognormal assumption is used, two other possible approaches exist. However, neither of these methods will be valid for most data sets with a large proportion of censored values and they should not be used without support from a qualified statistician.
The first method involves the use of nonparametric methods to calculate an upper 95% confidence limit. Since nonparametric methods are available for calculating an upper confidence limit on a percentile, but not the mean, it is necessary to estimate the percentile corresponding to the mean of the best-fit distribution, and then calculate the upper 95% confidence limit on that percentile (see Guidance Sections 52.2.3-5.22.4).
The second method involves the use of a probability plot to estimate the mean and standard deviation of the distribution (Gilbert 1987, p. 168), which can then be used to calculate the upper confidence limit on the mean.

Site data (cont.)

Decisions based on the largest value in the data set (Case 2, Part III; Case 3).

Use of the maximum value in the data set as the UCL can be a stringent test for compliance with a cleanup level. This is particularly true if the maximum is considerably higher than any other values in the data but does not violate the compliance criterion in the Cleanup Regulation of exceeding twice the cleanup level. Thus the appropriate decision when the maximum exceeds the cleanup level may be to conduct a further evaluation, rather than making a compliance decision. The following examples illustrate some possible approaches for a further evaluation:

1) Resampling with a lower detection limit to obtain uncensored data. If a lower detection limit cannot be achieved, is the maximum is confirmed by the second round of sampling?

2) If the data are from soil sampling, does the maximum corresponds to a hot spot that should be evaluated separately from the remaining sampling data? If it is a hot spot, the possible existence of additional hot spots should be considered.

3) If the data are for groundwater, review the sampling and analysis QA/OC procedures followed and examine the data for temporal trends or correlations with groundwater levels. For example, the review might reveal problems with the collection or analysis of the low-value samples or the high-concentration sample. As another example, the review might suggest that large variations in contaminant concentrations are associated with seasonal changes in groundwater level, and that additional sampling is needed to provide confirmation.

Background data sets: calculating a background value.

The procedures for establishing background described in the Guidance (Section 4.3.3) require the calculation of percentiles, or "quantiles", (e.g., 50th and 90th percentiles, for lognormal distributed data). In addition, the **coefficient of variation** is needed when a lognormal or normal distribution applies, to determine the allowable exceedance for a background-based cleanup standard (Guidance Figure 12, Section 4.3.3.2). For censored data sets, the recommended procedures for calculating these statistics are described below.

Summary Table

Percentage of non-detects or below-PQL values in data set	Recommended procedure	Discussion
Not more than 50%	*Lognormal distribution*: Use regression equation for probability plot with the log-transformed data to calculate the 50th and 90th percentiles. *Normal distribution*: Use regression equation for probability plot with the untransformed data to calculate the 50th and 80th percentiles. *Neither distribution*: Use nonparametric method for calculating a percentile (Guidance Example 5, Section 2.1.2.3).	Case 4
More than 50%	Use nonparametric method for calculating a percentile (Guidance Example 5, Section 2.1.2.3)	Case 5

Case 4. No more than 50% of the data are censored values (non-detects or below-PQL values).

PART I Default assumption is that data come from a lognormal distribution.

Step 1. Test the default assumption of a lognormal distribution using the normal probability plot procedure described on page 2. If the assumption is rejected, proceed to Part II.

Step 2. Calculate the 50th and 90th percentiles from the probability plot regression (page 3).

Step 3. If background value is for use as a cleanup standard, calculate the coefficient of variation from the linear regression equation (page 4).

Step 4. Follow procedures described in Guidance Section 4.33 (see Guidance Figure 12).

Background data (cont.)

PART II Data are not lognormally distributed.

Step 1. Test the default assumption of a normal distribution using the normal probability plot procedure described on page 2. If the assumption is rejected, proceed to Part III.

Step 2. Calculate the 50th and 80th percentiles from the probability plot regression (page 3).

Step 3. If background value is for use as a cleanup standard, calculate the coefficient of variation from the linear regression equation (page 4).

Step 4. Follow procedures described in Guidance Section 43.3 (see Guidance Figure 12).

PART III Data are neither lognormally nor normally distributed.

Use the nonparametric method for calculating a percentile selected as the background value (see Guidance Example 5, Section 2.1.2.3). Requires consultation with Ecology.

Case 5. More than 50% of the data are censored values.

Use the nonparametric method for calculating a percentile selected as the background value (see Guidance Example 5, Section 2.1.2.3). Requires consultation with Ecology (see Guidance, Section 1.2).

Notes: For some data sets, it may be possible to use a probability plot to make a decision on the data distribution, and to estimate percentiles, as in Case 5. However, this approach may often be statistically invalid.

Note 1. Additional comments

1) In some instances, data from replicate samples are reported as below the detection limit (or PQL) for one of the replicate results but not the other. The recommended solution is to assign half the detection limit to the censored value (or the MDL, for a below-PQL value) and use the average of this number and the uncensored value. Treat the average as an uncensored value. (Example: results are reported as 5.0 ppm for one replicate and not detected, at a detection limit of 3.0 ppm, for the other. Use 3.3 ppm as the single value from the replicates for statistical analyses.)

2) An acceptable and technically preferable method of assigning ranks to censored data for probability plot analyses is described by Hughes and Millard (1988). However, the method is difficult to implement in practice and is therefore not recommended for routine use.

3) Further information on the probability plot procedure described in this Supplement can be found in statistical textbooks (e.g., Snedocor and Cochran 1989, page 59).

4) Portions of the Model Toxics Control Act Cleanup Regulation (Chapter 173-340 WAC) dealing with censored data include the following:

Definitions of "method detection limit" and "practical quantitation limit" WAC 173-340-200
Analytical considerations WAC 1 73-340-707(2)-(4)
Simple substitution method and alternatives WAC 173-340-708(11), WAC 173-340-720(8),
WAC 173-340-730(7), WAC 173-340-740(7)
Sampling and analysis plans WAC 1 73-340-820(2)(d)
Analytical procedures WAC 173-340-830(2)-(4)

Acknowledgements. The method for analyzing probability plots described in this Supplement was suggested by Colin Wagoner (ICF Technology Incorporated). The Supplement was prepared with assistance from Greg Glass.

Note 2. Use of the largest value in a data set as an estimate of the upper 95% confidence limit on the mean.

Because the underlying distribution for sampling data is often positively skewed (e.g., Helsel 1990), the mean will generally correspond to an upper percentile (i.e., larger than the 50th percentile). When the nonparametric method described in Section 5.2.2.3 of the Guidance is used on percentiles (including whatever percentile above the 50th percentile is equivalent to the mean), the conditions under which the UCL will correspond to the maximum value from the data set can be determined from Guidance Table A-5. Similarly, boundary conditions from the method in Section 5.2.2.4 (i.e. for more than 20 samples) can be determined from the equation given there. These conditions are tabled below:

Sample Size	Approximate percentile at which UCL=maximum value	
6	60	From Guidance Table A-5
7	65	
8	70	
9	75	
10	75	
11	75	
12	80	
13	80	
14	80	
15	80	
16	80	
17	85	
18	85	
20	85	
25	85	From Guidance Section 5.2.2.4
30	87	
40	90	
50	92	

As an example, the table shows that the nonparametric upper 95% confidence limit on the 85th percentile is the maximum value in the data set when there are 20 samples. Use of the maximum for demonstrating compliance with a cleanup level will be a conservative approach (using nonparametric statistical methods) with 20 samples unless the distribution is so highly skewed that the mean exceeds the 85th percentile.

Note 3. Selecting a percentile for use with the nonparametric method for calculating a background value.

When background data for a specific substance are used for site cleanup decisions, the data must be reduced to a single background value which can then be used for compliance decisions, as described in the Guidance (Section 4.3.5). Under the procedures described in the Guidance, the background value is a percentile (quantile) of the background distribution. Once a percentile has been selected, its value can be estimated from the background data set

Ecology's approach for selecting a percentile emphasizes the importance of balancing Type I and II error rates for background-based compliance decisions. For the cases of a lognormal or normal distribution, Ecology used estimates of these rates to select the 90th and 80th percentiles, respectively, as background values (Guidance Section 4.3.3.2). In these analyses, expected Type I and II error rates were estimated using Monte Carlo simulations, under a range of conditions (coefficients of variation).

It is impractical to conduct similar analyses to select a percentile that should be used for all data sets that are neither lognormally or normally distributed. The appropriate distribution and distributional parameters (e.g., coefficient of variation) for such data cannot be anticipated, and will likely vary considerably from one data set to another. Thus, while the nonparametric method for calculating a percentile is recommended for these data, a particular percentile that should be used in every case has not been selected. The nonparametric method is also recommended for heavily censored background data sets (Case 5 in this Supplement), and here again there is no percentile to be used in every case.

For remedial actions where the requirement to consult with Ecology applies, a percentile will be selected on a case by case basis. Refer to Section 1.2 of the Guidance regarding the applicability of this requirement.

For persons conducting independent cleanups, the following recommendations on selecting a percentile may be helpful:

1) Where the nonparametric method is used because of the large proportion of censored data (Case 5 in this Supplement), assume that the default assumption of an underlying lognormal distribution applies unless there is obvious evidence to the contrary. This means that the nonparametric method would be used to estimate the 50th and 90th percentiles, and background would be set at the 90th or four times the 50th, whichever is lower.

2) In some cases, a background data set may deviate from a lognormal or normal distribution when it is obtained by pooling data from different statistical populations. As a simple example, suppose that iron concentrations from an iron-rich soil follow a normal distribution and concentrations from an iron-poor soil also follow a normal distribution, but one with a lower mean. Pooled data from the two soils might produce a bimodal, rather than a normal, distribution.

   As this example illustrates, it may be useful to examine data which do not fit either a lognormal or normal distribution to determine whether subsets of the data are compatible with one of these distributions. However, there should be a valid technical justification for forming subsets that may represent different statistical populations. Separating data from surface and subsurface soil samples may be valid, for example, but separating soil data solely on the basis of the date of sampling would not.

   If the data subsets fit a lognormal or normal distribution, it may be possible to calculate a background value for each subset, using the procedures for these distributions (Guidance Section 4.3.3.2). The relevance of the sample subsets to site conditions should be considered in deciding which background value will be used. For example, if the subsets correspond to different soil types, use the background value for the soil type that is most representative of site soils. If there is no basis for identifying the most relevant subset, an environmentally conservative approach would be to use the lowest background value calculated from the different subsets.

   A background value calculated from a subset of the data cannot be used for site cleanup decisions if the subset is too small. At a minimum, a subset of soil samples must include 10 measurements for natural background and at least 20 for area background EWAC
173-340-708(11)(d)].

3) When only a small number of background measurements are available (e.g , <20), a few anomalous values can greatly affect the interpretation of the data. Thus, there is a possibility that even when the underlying distribution is lognormal (or normal), this may not be apparent with a small data set. For this reason, the preferred approach for a small data set that does not fit either of these distributions is to conduct additional sampling, rather than proceeding with the nonparametric method for calculating a background value.

4) If a large data set (e.g., >50) resembles, but does not fit a lognormal distribution, use the smaller of the 90th percentile or four times the 50th percentile, as estimated with the nonparametric method.

   As an example, the data distribution may be more skewed to the right (i.e., have a more positive skew or longer "tail") than the fitted lognormal distribution. [On a normal probability plot, this means that the larger (log-transformed) values will curve up above a straight line fitted through all the points on the plot. However, a histogram of the data may nevertheless resemble the shape of a lognormal distribution (see Guidance Figure 4).

5) If these recommendations do not resolve the problem, use the Wilcoxon Rank Sum Test or other suitable nonparametric test to demonstrate compliance with a background-based cleanup standard. This approach does not require the calculation of a percentile from background data and does not require the comparison of an upper confidence limit on compliance data with a cleanup level. The test is used to show that contaminant concentrations, measured after cleanup, are not significantly different from concentrations at a background area. Use of this approach should be supported by a statistical power analysis, showing the exceedance of background which could be detected with high probability under the proposed sampling plan.

   The procedures for conducting the Wilcoxon Rank Sum Test (equivalent to the Mann-Whitney U Test) are described in Gilbert (1987; pages 247-250). Various statistical packages can be used to perform this test, which should not be confused with the Wilcoxon Signed Rank Test.

A final option is to seek assistance from a statistician in analyzing expected Type I and II error rates (Guidance Section 2.1.7) for a distribution fitted to the background data. Error rates should be estimated for several candidate percentiles (e.g., 80th, 90th and 95th). A reasonable range of values for distribution shape parameters (e.g. coefficient of variation) should also be included in the analysis. The distribution used for background and site data need not be the same if there are data to support the use of different distributions. A brief explanation of the approach for estimating Type I and II error rates is given in Guidance Section 4.3.3.2. Further information can be obtained from Ecology.

Documentation from the analysis should show that the selected percentile provides a reasonable balance between expected Type I and II error rates. The estimated Type I error rate should decline as the level of site contamination increases above the background distribution. If it has not declined to at least 0.05 at the level of site contamination which corresponds to a doubling of risk, the selected percentile is unlikely to be acceptable to Ecology, and a lower percentile should be considered. The requirement for a Type I error rate of 0.05 is found in the MTCA Cleanup Regulation [e.g., WAC 173-340-720(8)(e)(i)].

BIBLIOGRAPHY

Cohen, A.C., Jr. 1959. Simplified estimators for the normal distribution when samples are singly censored or truncated. Technometrics 1:217-237.

Cohen, A.C., Jr. 1961. Tables for maximum likelihood estimates: singly truncated and singly censored samples. Technometrics 3:535-541.

Gilbert, R.O. 1987. Statistical methods for environmental pollution monitoring. Van Nostrand Reinhold Company, New York, NY. 320 pp.

Helsel, D.R. 1990. Less than obvious: Statistical treatment of data below the detection limit. Environmental Science and Technology 24:1766-1774.

Hughes, J.P. and S.P. Millard. 1988. A tau-like test for trend in the presence of multiple censoring points. Water Resources Bulletin 24:521-531.

Snedecor, G.W. and W.G. Cochran. 1989. Statistical methods. Eighth edition. Iowa State University Press, Ames. 503 pp.

U.S. EPA. 1989. Statistical analysis of ground-water monitoring data at RCRA facilities. Interim final guidance. U.S. Environmental Protection Agency Office of Solid Waste, Washington, DC.

U.S. EPA. 1992. Statistical analysis of ground-water monitoring data at RCRA facilities. Addendum to interim final guidance. U.S. Environmental Protection Agency Office of Solid Waste, Washington, DC.

WORKSHEETS

WORKSHEET W-4 Cohen's Method for Calculating Adjusted Mean and Standard Deviation of Censored Data (Lognormal Distribution)

Col. 1	Col. 2
Uncensored data (need not be sorted)	Log - transformed

Calculations:

T		Average of values in Col. 2
Ś		Standard deviation (Col. 2)
V		Value in Box Ś squared (= variance)
C		Number of censored values in data set
N		Number of samples (censored and uncensored)
Δ		T – Ln(X) where X = detection limit or PQL
h		C/N
γ		V/Δ^2
λ		Use h and γ to find a value for λ in Table A-8 (attached)
M		T – (λ x Δ) (adjusted mean)
S		$\sqrt{[V + (\lambda \times \Delta^2)]}$ (adjusted std. devn.)

To calculate the upper confidence limit on the mean, enter the values from Box M and S in the corresponding boxes on Worksheet W-2 and Complete that worksheet

† Uncensored data are the values above the detection Limit or PQL

WORKSHEET W-4a Cohen's Method for Calculating Adjusted Mean and Standard Deviation of Censored Data (Normal Distribution)

Col. 1
Uncensored data (need not be sorted)

Calculations:

T		Average of values in Col. 1
Ś		Standard deviation (Col. 1)
V		Value in Box Ś squared (= variance)
C		Number of censored values in data set
N		Number of samples (censored and uncensored)
Δ		T – X where X = detection limit or PQL
h		C/N
γ		V/Δ^2
λ		Use h and γ to find a value for λ in Table A-8 (attached)
M		T – (λ x Δ) (adjusted mean)
S		$\sqrt{[V + (\lambda \times \Delta^2)]}$ (adjusted std. dvn.)

Calculation of upper 95% confidence limit:

df		N – 1
T		Use df to obtain t (see Guidance Table A-4 at α=.05)
UCL		$M + [(t \times S)/\sqrt{N}]$

TABLE A-8. VALUES OF λ FOR USE WITH WORKSHEETS W-4 AND W-4a

Use the values for h and γ calculated from the worksheet to find a value for λ, which is needed in subsequent worksheet calculations. When the table does not contain the exact entries for h and γ, double linear interpolation should be used to estimate λ. The following, adapted from EPA (1989, p. 8-10), illustrates the procedure:

Example calculation. Assume that values of h = 0.18 and γ = 1.31 have been obtained from worksheet calculations. Since these values do not appear in Table A-8, λ musts be found by double linear interpolation.

The values from the table which are needed ford the interpolation are:

γ	h = 0.15	h = 0.20
1.30	0.26610	0.36610
1.35	0.26860	0.36950

There are 0.03 units between 0.15 and 0.18 on the h-scale. There are 0.05 units between 0.15 and 0.20. Therefore, the value of interest (0.18) lies (0.03/0.05 * 100) = 60% of the distance along the interval between 0.15 and 0.20. To linearly interpolate between the tabulated values on the h axis, the range between the values must be calculated, the value which is 60% of the distance along the range must be computed and then that value must be added to the lower point on the tabulated values. The result is the interpolated value. The interpolated points on the h-scale for the current example are:

At γ=1.30   0.36610 - 0.26610 = 0.10000   0.10000 * 0.60 = 0.06000
0.26610 + 0.06000 = <u>0.32610</u>

At γ=1.35   0.36950 – 0.26860 = 0.10090   0.10090 * 0.60 = 0.06054
0.26860 + 0.06054 = <u>0.32914</u>

On the γ-axis there are 0.01 units between 1.30 and 1.31. There are 0.05 units between 1.30 and 1.35. The value of interest (1.31) lies (0.01/0.05 * 100) = 20% of the distance along the interval between 1.30 and 1.35. The interpolated point on the γ-axis is:

0.32914 – 0.32610 = 0.00304   0.00304 * 0.20 = 0.000608
0.32610 + 0.000608 = <u>0.32671</u>

Thus, λ = 0.32671.

TABLE A-8. Values for λ

(Work sheets W-4 and W-4a)

Source: EPA (1989, 1992)

Proportion of censored values in data set (h)

γ	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.17342	0.24268	0.31362	0.40210	0.49410	0.59610	0.70960	0.83680
0.01	0.17470	0.24430	0.32050	0.40430	0.49670	0.59890	0.71280	0.84030
0.05	0.17935	0.25033	0.32793	0.41300	0.50660	0.61010	0.72520	0.85400
0.10	0.18479	0.25741	0.33662	0.42330	0.51840	0.62340	0.74000	0.87030
0.15	0.18985	0.26405	0.34480	0.43300	0.52960	0.63610	0.75420	0.88600
0.20	0.19460	0.27031	0.35255	0.44220	0.54030	0.64830	0.76780	0.90120
0.25	0.19910	0.27626	0.35993	0.45100	0.55060	0.66000	0.78100	0.91580
0.30	0.20338	0.28193	0.36700	0.45950	0.56040	0.67130	0.79370	0.93000
0.35	0.20747	0.28737	0.37379	0.46760	0.56990	0.68210	0.80600	0.94370
0.40	0.21139	0.29260	0.38033	0.47550	0.57910	0.69270	0.81790	0.95700
0.45	0.21517	0.29765	0.38665	0.48310	0.58800	0.70290	0.82950	0.97000
0.50	0.21882	0.30253	0.39276	0.49040	0.59670	0.71290	0.84080	0.98260
0.55	0.22235	0.30725	0.39870	0.49760	0.60510	0.72250	0.85170	0.99500
0.60	0.22578	0.31184	0.40447	0.50450	0.61330	0.73200	0.86250	1.00700
0.65	0.22910	0.31630	0.41008	0.51140	0.62130	0.74120	0.87290	1.01880
0.70	0.23234	0.32065	0.41555	0.51800	0.62910	0.75020	0.88320	1.03030
0.75	0.23550	0.32489	0.42090	0.52450	0.63670	0.75900	0.89320	1.04160
0.80	0.23858	0.32903	0.42612	0.53080	0.64410	0.76760	0.90310	1.05270
0.85	0.24158	0.33307	0.43122	0.53700	0.65150	0.77610	0.91270	1.06360
0.90	0.24452	0.33703	0.43622	0.54300	0.65860	0.78440	0.92220	1.07430
0.95	0.24740	0.34091	0.44112	0.54900	0.66560	0.79250	0.93140	1.08470
1.00	0.25022	0.34471	0.44592	0.55480	0.67240	0.80050	0.94060	1.09510
1.05	0.25300	0.34840	0.45060	0.56050	0.67930	0.80840	0.94960	1.10520
1.10	0.25570	0.35210	0.45530	0.56620	0.68600	0.81610	0.95840	1.11520
1.15	0.25840	0.35570	0.45980	0.57170	0.69250	0.82370	0.96710	1.12500
1.20	0.26100	0.35920	0.46430	0.57710	0.69900	0.83120	0.97560	1.13470
1.25	0.26360	0.36270	0.46870	0.58250	0.70530	0.83850	0.98410	1.14430
1.30	0.26610	0.36610	0.47300	0.58780	0.71150	0.84580	0.99240	1.15370
1.35	0.26860	0.36950	0.47730	0.59300	0.71770	0.85290	1.00060	1.16290
1.40	0.27100	0.37280	0.48150	0.59810	0.72380	0.86000	1.00870	1.17210
1.45	0.27350	0.37610	0.48560	0.60310	0.72980	0.86700	1.01660	1.18120
1.50	0.27580	0.37930	0.48970	0.60810	0.73570	0.87380	1.02450	1.19010
1.55	0.27820	0.38250	0.49380	0.61300	0.74150	0.88060	1.03230	1.19890
1.60	0.28050	0.38560	0.49770	0.61790	0.74720	0.88730	1.04000	1.20760
1.65	0.28280	0.38870	0.50170	0.62270	0.75290	0.89390	1.04760	1.21620
1.70	0.28510	0.39180	0.50550	0.62740	0.75850	0.90050	1.05510	1.22480
1.75	0.28730	0.39480	0.50940	0.63210	0.76410	0.90690	1.06250	1.23320
1.80	0.28950	0.39780	0.51320	0.63670	0.76960	0.91330	1.06980	1.24150
1.85	0.29170	0.40070	0.51690	0.64130	0.77500	0.91960	1.07710	1.24970
1.90	0.29380	0.40360	0.52060	0.64580	0.78040	0.92590	1.08420	1.25790
1.95	0.29600	0.40650	0.52430	0.65020	0.78570	0.93210	1.09130	1.26600
2.00	0.29810	0.40930	0.52790	0.65470	0.79090	0.93820	1.09840	1.27390
2.05	0.30010	0.41220	0.53150	0.65900	0.79610	0.94420	1.10530	1.28190
2.10	0.30220	0.41490	0.53500	0.66340	0.80130	0.95020	1.11220	1.28970
2.15	0.30420	0.41770	0.53850	0.66760	0.80630	0.95620	1.11900	1.29740
2.20	0.30620	0.42040	0.54200	0.67190	0.81140	0.96200	1.12580	1.30510
2.25	0.30820	0.42310	0.54540	0.67610	0.81640	0.96790	1.13250	1.31270
2.30	0.31020	0.42580	0.54880	0.68020	0.82130	0.97360	1.13910	1.32030
2.35	0.31220	0.42850	0.55220	0.68440	0.82620	0.97940	1.14570	1.32780
2.40	0.31410	0.43110	0.55550	0.68840	0.83110	0.98500	1.15220	1.33520
2.45	0.31600	0.43370	0.55880	0.69250	0.83590	0.99060	1.15870	1.34250
2.50	0.31790	0.43630	0.56210	0.69650	0.84070	0.99620	1.16510	1.34980
2.55	0.31980	0.43880	0.56540	0.70050	0.84540	1.00170	1.17140	1.35710
2.60	0.32170	0.44140	0.56860	0.70440	0.85010	1.00720	1.17770	1.36420
2.65	0.32360	0.44390	0.57180	0.70830	0.85480	1.01260	1.18400	1.37140
2.70	0.32540	0.44640	0.57500	0.71220	0.85940	1.01800	1.19020	1.37840
2.75	0.32720	0.44890	0.57810	0.71610	0.86390	1.02340	1.19630	1.38540
2.80	0.32900	0.45130	0.58120	0.71990	0.86850	1.02870	1.20240	1.39240
2.85	0.33080	0.45370	0.58430	0.72370	0.87300	1.03390	1.20850	1.39930
2.90	0.33260	0.45620	0.58740	0.72740	0.87750	1.03920	1.21450	1.40610
2.95	0.33440	0.45850	0.59050	0.73110	0.88190	1.04430	1.22050	1.41290

TABLE A-8. Values for λ (Cont.)

Proportion of censored values in data set (h)

γ	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
3.00	0.33610	0.46090	0.59350	0.73480	0.88630	1.04950	1.22640	1.41970
3.05	0.33780	0.46330	0.59650	0.73850	0.89070	1.05460	1.23230	1.42640
3.10	0.33960	0.46560	0.59950	0.74220	0.89500	1.05970	1.23810	1.43300
3.15	0.34130	0.46790	0.60240	0.74580	0.89930	1.06470	1.24390	1.43960
3.20	0.34300	0.47030	0.60540	0.74940	0.90360	1.06970	1.24970	1.44620
3.25	0.34470	0.47250	0.60830	0.75290	0.90790	1.07470	1.25540	1.45270
3.30	0.34640	0.47480	0.61120	0.75650	0.91210	1.07960	1.26110	1.45920
3.35	0.34800	0.47710	0.61410	0.76000	0.91630	1.08450	1.26680	1.46570
3.40	0.34970	0.47930	0.61690	0.76350	0.92050	1.08940	1.27240	1.47200
3.45	0.35130	0.48160	0.61970	0.76700	0.92460	1.09420	1.27790	1.47840
3.50	0.35290	0.48380	0.62260	0.77040	0.92870	1.09900	1.28350	1.48470
3.55	0.35460	0.48600	0.62540	0.77390	0.93280	1.10380	1.28900	1.49100
3.60	0.35620	0.48820	0.62820	0.77730	0.93690	1.10860	1.29450	1.49720
3.65	0.35780	0.49030	0.63090	0.78070	0.94090	1.11330	1.29990	1.50340
3.70	0.35940	0.49250	0.63370	0.78400	0.94490	1.11800	1.30530	1.50960
3.75	0.36090	0.49460	0.63640	0.78740	0.94890	1.12260	1.31070	1.51570
3.80	0.36250	0.49680	0.63910	0.79070	0.95290	1.12730	1.31600	1.52180
3.85	0.36410	0.49890	0.64180	0.79400	0.95680	1.13190	1.32130	1.52790
3.90	0.36560	0.50100	0.64450	0.79730	0.96070	1.13640	1.32660	1.53390
3.95	0.36720	0.50310	0.64720	0.80060	0.96460	1.14100	1.33180	1.53990
4.00	0.36870	0.50520	0.64980	0.80380	0.96850	1.14550	1.33710	1.54580
4.05	0.37020	0.50720	0.65250	0.80700	0.97230	1.15000	1.34230	1.55180
4.10	0.37170	0.50930	0.65510	0.81020	0.97620	1.15450	1.34740	1.55770
4.15	0.37320	0.51130	0.65770	0.81340	0.98000	1.15900	1.35260	1.56350
4.20	0.37470	0.51340	0.66030	0.81660	0.98370	1.16340	1.35770	1.56930
4.25	0.37620	0.51540	0.66290	0.81980	0.98750	1.16780	1.36270	1.57510
4.30	0.37770	0.51740	0.66540	0.82290	0.99130	1.17220	1.36780	1.58090
4.35	0.37920	0.51940	0.66800	0.82600	0.99500	1.17650	1.37280	1.58660
4.40	0.38060	0.52140	0.67050	0.82910	0.99870	1.18090	1.37780	1.59240
4.45	0.38210	0.52340	0.67300	0.83220	1.00240	1.18520	1.38280	1.59800
4.50	0.38360	0.52530	0.67550	0.83530	1.00600	1.18950	1.38780	1.60370
4.55	0.38500	0.52730	0.67800	0.83840	1.00970	1.19370	1.39270	1.60930
4.60	0.38640	0.52920	0.68050	0.84140	1.01330	1.19800	1.39760	1.61490
4.65	0.38790	0.53120	0.68300	0.84450	1.01690	1.20220	1.40240	1.62050
4.70	0.38930	0.53310	0.68550	0.84750	1.02050	1.20640	1.40730	1.62600
4.75	0.39070	0.53500	0.68790	0.85050	1.02410	1.21060	1.41210	1.63150
4.80	0.39210	0.53700	0.69030	0.85350	1.02770	1.21480	1.41690	1.63700
4.85	0.39350	0.53890	0.69280	0.85640	1.03120	1.21890	1.42170	1.64250
4.90	0.39490	0.54070	0.69520	0.85940	1.03480	1.22300	1.42650	1.64790
4.95	0.39630	0.54260	0.69760	0.86230	1.03830	1.22720	1.43120	1.65330
5.00	0.39770	0.54450	0.70000	0.86530	1.04180	1.23120	1.43590	1.65870
5.05	0.39900	0.54640	0.70240	0.86820	1.04520	1.23530	1.44060	1.66410
5.10	0.40040	0.54820	0.70470	0.87110	1.04870	1.23940	1.44530	1.66940
5.15	0.40180	0.55010	0.70710	0.87400	1.05210	1.24340	1.45000	1.67470
5.20	0.40310	0.55190	0.70940	0.87680	1.05560	1.24740	1.45460	1.68000
5.25	0.40450	0.55370	0.71180	0.87970	1.05900	1.25140	1.45920	1.68530
5.30	0.40580	0.55560	0.71410	0.88250	1.06240	1.25540	1.46380	1.69050
5.35	0.40710	0.55740	0.71640	0.88540	1.06580	1.25940	1.46840	1.69580
5.40	0.40850	0.55920	0.71870	0.88820	1.06910	1.26330	1.47290	1.70100
5.45	0.40980	0.56100	0.72100	0.89100	1.07250	1.26720	1.47750	1.70610
5.50	0.41110	0.56280	0.72330	0.89380	1.07580	1.27110	1.48200	1.71130
5.55	0.41240	0.56460	0.72560	0.89660	1.07920	1.27500	1.48650	1.71640
5.60	0.41370	0.56630	0.72780	0.89940	1.08250	1.27890	1.49100	1.72150
5.65	0.41500	0.56810	0.73010	0.90220	1.08580	1.28280	1.49540	1.72660
5.70	0.41630	0.56990	0.73230	0.90490	1.08910	1.28660	1.49990	1.73170
5.75	0.41760	0.57160	0.73460	0.90770	1.09240	1.29050	1.50430	1.73680
5.80	0.41890	0.57340	0.73680	0.91040	1.09560	1.29430	1.50870	1.74180
5.85	0.42020	0.57510	0.73900	0.91310	1.09890	1.29810	1.51310	1.74680
5.90	0.42150	0.57690	0.74120	0.91580	1.10210	1.30190	1.51750	1.75180
5.95	0.42270	0.57860	0.74340	0.91850	1.10530	1.30570	1.52180	1.75680
6.00	0.42400	0.58030	0.74560	0.92120	1.10850	1.30940	1.52620	1.76170

TABLE A-8 PAGE 2

WORKSHEETS

WORKSHEET W-1 Calculations for W test.

Col. 1 Data sorted lowest to highest	Col. 2 Log_e-transformed	Col. 3 Numbers in Col. 2 squared

Calculations:

A		Col. 3 total
B		Col. 2 total
C		B^2
N		Number of samples
D		A - (C/N)
r		N/2 (N even) (N-1)/2 (N odd)

Proceed to Page 2 of Worksheet.

This worksheet shows calculations required to test the possibility that the data differ significantly from a **lognormal distribution** (see Section 2.1.4.2). The computation method shown here differs slightly from that shown in Example 7 to avoid rounding errors that give a less accurate value for W.

If the data are not lognormally distributed, Worksheet W-1a shows the calculations required to test the possibility that the data differ significantly from a **normal distribution** (see Section 2.1.4.1).

See Supplement S-3 for further information, including alternatives to the use of these Worksheets.

WORKSHEET W-1.

Col. 4 List the <u>first</u> r numbers from Col. 2 (smallest to largest)	Col. 5 List the <u>last</u> r numbers from Col. 2 (largest to smallest)	Col. 6 List the coefficients from Table A-1	Col. 7 (Col. 5 - Col. 4) X Col. 6

Calculations:

E		Col. 7 total
F		E^2
W		**F/D**

Compare **W** with the appropriate value below. If **W** is <u>smaller</u> than the tabled value, the default assumption of a lognormal distribution for the data must be rejected.

Sample size (N)	Critical value	Sample size (N)	Critical value
3	0.767	27	0.923
4	0.748	28	0.924
5	0.762	29	0.926
6	0.788	30	0.927
7	0.803	31	0.929
8	0.818	32	0.930
9	0.829	33	0.931
10	0.842	34	0.933
11	0.850	35	0.934
12	0.859	36	0.935
13	0.866	37	0.936
14	0.874	38	0.938
15	0.881	39	0.939
16	0.887	40	0.940
17	0.892	41	0.941
18	0.897	42	0.942
19	0.901	43	0.943
20	0.905	44	0.944
21	0.908	45	0.945
22	0.911	46	0.945
23	0.914	47	0.946
24	0.916	48	0.947
25	0.918	49	0.947
26	0.920	50	0.947

WORKSHEET W-1 Calculations for W test.

EXAMPLE

Col. 1 Data sorted lowest to highest	Col. 2 Log_e-transformed	Col. 3 Numbers in Col. 2 squared
0·13	-2.042	4.163
0·45	-0·7985	0.638
0·60	-0·5108	0.261
0·76	-0·2744	0.075
1.05	0·0488	0.002
1.12	0·1133	0·013
1.20	0·1823	0·033
1.37	0·3148	0.099
1.69	0·5247	0.275
2.06	0·7227	0.522

Calculations:

A	6.0816	Col. 3 total
B	-1.7173	Col. 2 total
C	2·9491	B^2
N	10	Number of samples
D	5.7867	A - (C/N)
r	5	N/2 (N even) (N-1)/2 (N odd)

Proceed to Page 2 of Worksheet.

This worksheet shows calculations required to test the possibility that the data differ significantly from a **lognormal distribution** (see Section 2.1.4.2). The computation method shown here differs slightly from that shown in Example 7 to avoid rounding errors that give a less accurate value for W.

If the data are not lognormally distributed, Worksheet W-1a shows the calculations required to test the possibility that the data differ significantly from a **normal distribution** (see Section 2.1.4.1).

See Supplement S-3 for further information, including alternatives to the use of these Worksheets.

 ↓ coefficients for N=10

r=5 from page 1

Col. 4 — List the first r numbers from Col. 2 (smallest to largest)	Col. 5 — List the last r numbers from Col. 2 (largest to smallest)	Col. 6 — List the coefficients from Table A-1	Col. 7 — (Col. 5 - Col. 4) X Col. 6
-2.0402	0.7227	0.5739	1.5856
-0.7985	0.5247	0.3291	0.4355
-0.5108	0.3148	0.2141	0.1768
-0.2744	0.1823	0.1224	0.0559
0.0488	0.1133	0.0399	0.0026

Calculations:

E	2.2564	Col. 7 total
F	5.0912	E^2
W	0.8798	F/D

Compare **W** with the appropriate value below. If **W** is smaller than the tabled value, the default assumption of a lognormal distribution for the data must be rejected.

0.8798 is larger than 0.842, so can't reject assumption of lognormal distribution.

Sample size (N)	Critical value	Sample size (N)	Critical value
3	0.767	27	0.923
4	0.748	28	0.924
5	0.762	29	0.926
6	0.788	30	0.927
7	0.803	31	0.929
8	0.818	32	0.930
9	0.829	33	0.931
10	0.842	34	0.933
11	0.850	35	0.934
12	0.859	36	0.935
13	0.866	37	0.936
14	0.874	38	0.938
15	0.881	39	0.939
16	0.887	40	0.940
17	0.892	41	0.941
18	0.897	42	0.942
19	0.901	43	0.943
20	0.905	44	0.944
21	0.908	45	0.945
22	0.911	46	0.945
23	0.914	47	0.946
24	0.916	48	0.947
25	0.918	49	0.947
26	0.920	50	0.947

WORKSHEET W-1a. Calculations for W test
(Use this worksheet only if lognormal distribution has been rejected).

Col. 1 Data sorted lowest to highest	Col. 2 Numbers in Col. 1 squared

Calculations:

A		Col. 2 total
B		Col. 1 total
C		B^2
N		Number of samples
D		A - (C/N)
r		N/2 (N even) (N-1)/2 (N odd)

Proceed to Page 2 of Worksheet.

This worksheet shows calculations required to test the possibility that the data differ significantly from a **normal distribution** (see Section 2.1.4.1). The test should only be conducted if the default assumption of a lognormal distribution has been rejected. The computation method shown here differs slightly from that shown in Example 7 to avoid rounding errors that give a less accurate value for W.

See Supplement S-3 for further information, including alternatives to the use of these Worksheets.

WORKSHEET W-1a.

Col. 4	Col. 5	Col. 6	Col. 7
List the <u>first</u> **r** numbers from Col. 1 (smallest to largest)	List the <u>last</u> **r** numbers from Col. 1 (largest to smallest)	List the coefficients from Table A-1	(Col. 5 - Col. 4) X Col. 6

Calculations:

E		Col. 7 total
F		E^2
W		**F/D**

Compare **W** with the appropriate value below. If **W** is <u>smaller</u> than the tabled value, the assumption of a normal distribution for the data must be rejected.

Sample size (N)	Critical value	Sample size (N)	Critical value
3	0.767	27	0.923
4	0.748	28	0.924
5	0.762	29	0.926
6	0.788	30	0.927
7	0.803	31	0.929
8	0.818	32	0.930
9	0.829	33	0.931
10	0.842	34	0.933
11	0.850	35	0.934
12	0.859	36	0.935
13	0.866	37	0.936
14	0.874	38	0.938
15	0.881	39	0.939
16	0.887	40	0.940
17	0.892	41	0.941
18	0.897	42	0.942
19	0.901	43	0.943
20	0.905	44	0.944
21	0.908	45	0.945
22	0.911	46	0.945
23	0.914	47	0.946
24	0.916	48	0.947
25	0.918	49	0.947
26	0.920	50	0.947

WORKSHEET W-1a. Calculations for W test
(Use this worksheet only if lognormal distribution has been rejected).

Col. 1 Data sorted lowest to highest	Col. 2 Numbers in Col. 1 squared
5.14	26.40
12.68	160.86
13.62	185.61
19.98	399.23
36.08	1301.80
38.75	1501.83
42.88	1838.69
45.19	2041.86
45.22	2045.00
48.59	2360.65
52.33	2738.63
56.70	3214.44
62.13	3860.56
68.85	4740.61
80.62	6500.25
80.86	6538.50
86.56	7493.05
104.35	10889.34
104.45	10908.97
105.23	11072.30
114.15	13029.99
125.74	15811.30
149.32	22297.36
149.58	22375.07
150.54	22661.69
210.18	44176.05
334.51	111898.95
367.75	135242.27
377.49	142495.68
418.63	175253.59

EXAMPLE

Calculations:

A	785 060.5	Col. 2 total
B	3 508.1	Col. 1 total
C	12306915	B^2
N	30	Number of samples
D	374 830	A - (C/N)
r	15	N/2 (N even) (N-1)/2 (N odd)

Proceed to Page 2 of Worksheet.

This worksheet shows calculations required to test the possibility that the data differ significantly from a **normal distribution** (see Section 2.1.4.1). The test should only be conducted if the default assumption of a lognormal distribution has been rejected. The computation method shown here differs slightly from that shown in Example 7 to avoid rounding errors that give a less accurate value for W.

See Supplement S-3 for further information, including alternatives to the use of these Worksheets.

Col. 4 List the first r numbers from Col. 1 (smallest to largest)	Col. 5 List the last r numbers from Col. 1 (largest to smallest)	Col. 6 List the coefficients from Table A-1	Col. 7 (Col. 5 - Col. 4) X Col. 6
5.14	418.63	0.4254	175.9
12.68	377.49	0.2944	107.4
13.62	367.75	0.2487	88.1
19.98	334.51	0.2148	67.6
36.08	210.18	0.1870	32.6
38.75	150.54	0.1630	18.2
42.88	149.58	0.1415	15.1
45.19	149.32	0.1219	12.7
45.22	125.74	0.1036	8.3
48.59	114.15	0.0862	5.7
52.33	105.23	0.0697	3.7
56.70	104.45	0.0537	2.6
62.13	104.35	0.0381	1.6
68.85	86.56	0.0227	0.4
80.62	80.86	0.0076	0.002

Calculations:

E	539.8	Col. 7 total
F	291339.7	E^2
W	0.78	F/D

Compare **W** with the appropriate value below. If **W** is smaller than the tabled value, the assumption of a normal distribution for the data must be rejected.

Since 0.78 is smaller than 0.927, reject normal distribution.

Sample size (N)	Critical value	Sample size (N)	Critical value
3	0.767	27	0.923
4	0.748	28	0.924
5	0.762	29	0.926
6	0.788	30	0.927
7	0.803	31	0.929
8	0.818	32	0.930
9	0.829	33	0.931
10	0.842	34	0.933
11	0.850	35	0.934
12	0.859	36	0.935
13	0.866	37	0.936
14	0.874	38	0.938
15	0.881	39	0.939
16	0.887	40	0.940
17	0.892	41	0.941
18	0.897	42	0.942
19	0.901	43	0.943
20	0.905	44	0.944
21	0.908	45	0.945
22	0.911	46	0.945
23	0.914	47	0.946
24	0.916	48	0.947
25	0.918	49	0.947
26	0.920	50	0.947

WORKSHEET W-2 Calculations for Upper 95% Confidence Limit (UCL) using H-statistic (lognormally distributed data) - Land's Method

Col. 1 Data (need not be sorted)	Col. 2 Log_e-transformed

Calculations:

M		Average of values in Col. 2
S		Standard deviation (Col. 2)
N		Number of samples
P		$\sqrt{(N-1)}$
T		$0.5S^2$
H		Use S and N to find value for H in Figure A-1. See Supplement S-2 for a more extensive nomograph.
V		(S X H)/P
K		M + T + V
UCL		Exp(K) or e^K

See Section 5.2.1.2 and Example 11

WORKSHEET W-2 Calculations for Upper 95% Confidence Limit (UCL) using H-statistic (lognormally distributed data) - Land's Method

EXAMPLE

Col. 1 Data (need not be sorted)	Col. 2 Log_e-transformed
77.46	4.350
81.48	4.400
87.27	4.469
90.01	4.500
90.27	4.503
90.56	4.506
94.91	4.553
98.27	4.588
99.73	4.602
101.79	4.623
102.73	4.632
110.26	4.703
112.74	4.725
113.57	4.732
114.24	4.738
114.47	4.740
125.10	4.829
129.46	4.863
140.13	4.943
143.47	4.966

Calculations:

M	4.648	Average of values in Col. 2
S	0.172	Standard deviation (Col. 2)
N	20	Number of samples
P	4.359	$\sqrt{(N-1)}$
T	0.0148	$0.5S^2$
H	≈ 1.76	Use **S** and **N** to find value for **H** in Figure A-1. See Supplement S-2 for a more extensive nomograph.
V	0.069	**(S X H)/P**
K	4.732	**M + T + V**
UCL	113.6	Exp(K) or e^K

See Section 5.2.1.2 and Example 11

WORKSHEET W-3. Calculating a Background Value (BV) for lognormally distributed data (see Supplement S-4).

Col. 1 Data (need not be sorted)	Col. 2 Log_e-transformed

Calculations:

M		Average of numbers in Col. 2
S		Standard devn. (Col.2)
P		**M** + (1.282 × **S**)
$\mathbf{P_{90}}$		Exp(**P**) or $e^{\mathbf{P}}$ (P_{90} = 90th percentile)
$\mathbf{P_{50}}$		Exp(**M**) or $e^{\mathbf{M}}$ (P_{50} = 50th percentile or median)
$\mathbf{4P_{50}}$		4 × $\mathbf{P_{50}}$
BV		$\mathbf{P_{90}}$ or $\mathbf{4P_{50}}$ (if $\mathbf{P_{90}} > \mathbf{4P_{50}}$)

Note: If more than 15% of the data are values assigned to BDL or below-PQL measurements, use the nonparametric method (see Example 10) or use a probability plot, with numbers from Col. 2 (see Example 4, p. 77). For more information, see Section 2.2.1.

WORKSHEET W-3. Calculating a Background Value (BV) for lognormally distributed data (see Supplement S-4).

EXAMPLE

Col. 1 Data (need not be sorted)	Col. 2 Log_e-transformed
100.36	4.609
101.86	4.624
120.53	4.792
107.57	4.678
90.64	4.507
107.14	4.674
114.45	4.740
112.49	4.723
107.46	4.677
95.30	4.557

Calculations:

M	4.66	Average of numbers in Col. 2
S	0.086	Standard devn. (Col.2)
P	4.768	M + (1.282 × S)
$\mathbf{P_{90}}$	117.7	Exp(P) or e^P (P_{90} = 90th percentile)
$\mathbf{P_{50}}$	105.43	Exp(M) or e^M (P_{50} = 50th percentile or median)
$\mathbf{4P_{50}}$	421.7	4 × $\mathbf{P_{50}}$
BV	117.7	$\mathbf{P_{90}}$ or $\mathbf{4P_{50}}$ (if $\mathbf{P_{90}} > \mathbf{4P_{50}}$)

Note: If more than 15% of the data are values assigned to BDL or below-PQL measurements, use the nonparametric method (see Example 10) or use a probability plot, with numbers from Col. 2 (see Example 4, p. 77). For more information, see Section 2.2.1.